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Investigating the behavior of smart thin beams with piezoelectric actuators under dynamic loads

Ali Nikkhoo*

Department of Civil Engineering, University of Science and Culture, P.O. Box 13145-871, Tehran, Iran

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ABSTRACT

In this paper, the constitutive equation of motion for an Euler–Bernoulli beam in which a number of piezoelectric patches are bonded to the bottom and top surfaces of it, and arbitrary boundary conditions, is derived by employing Hamilton's principle. Assuming a number of linear springs with high stiffness as intermediate supports, the motion equation of a multi-span smart beam could be found. Classical linear optimal control algorithm with displacement–velocity and velocity–acceleration feedbacks is used. Utilizing eigenfunction expansion method, the equation of motion is decoupled into a number of ordinary differential equations. All the numerical examples are presented for the simple boundary conditions. The applied dynamic excitations are a rectangular impulse, moving load and the moving mass. Parametric studies on the capability of the control system in vibration suppression of the beams under these dynamic loads are achieved. The obtained results reveal the efficiency of the proposed control system in reducing the response of the beam structures to the required levels.

1. Introduction

Vibration suppression of civil and mechanical structures experiencing dynamic loads has received great attention by the research communities during the last decades. In this regard, different methods, including active, passive, semi-active and hybrid vibration control systems have been developed. However, the active techniques are proved to be more efficient rather than the passive ones if designed based on a robust and stable algorithm. A comprehensive review of various aspects and studies of active structural control has been provided by Korkmaz [1]. On the other hand, an actively controlled structure has the ability to sense, diagnose and actuate any external excitations. Therefore, the specialists have adopted the term smart structures almost two decades ago referring a certain extraordinary ability of structures or structural components in performing their design function [2]. Hurlebaus and Gaul [3] presented an extensive survey of the researches pertinent to the dynamics of smart structures. They reported miscellaneous applications of smart materials in enhancing the overall behavior of a broad range of structures against any dynamic loadings.

The early study on the application of piezoelectric materials as surface-bonded or embedded sensors or actuators for the shape control of beam structures under static or dynamic loads was carried out by Crawley and de Luis [4]. This concept has been investigated and extended afterward for a variety of structures by many researchers [5]. Concerning the application of the active piezoelectric materials in vibration control of civil structures, Song et al. [6] provided a valuable overview of this issue. They mentioned the utilization of the piezoceramic materials as sensors or actuators in different structures such as

* Tel.: +98 21 44252045; fax: +98 21 44214750.
E-mail address: nikkhoo@usc.ac.ir

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beams and trusses as well as frame and cable-stayed structures for vibration suppression purposes. They also affirmed that the light-weight, low-cost and easy implementation of such materials to be their main advantages for being widely used. Piezoelectric materials are used as patches bonded to the structure surfaces, stacks as axially actuators, or they could be used as an integrated layer in the composite structures. Sung [7] modeled a simply supported beam with two piezoelectric actuators bonded to the beam's bottom surface and a traversing moving mass as the external excitation. Utilizing a full-state feedback controller, the optimal placement of the piezo actuators was determined by the LQR-based formulation. He proved the performance of the control system to be acceptable even for high moving mass velocities. Rofooei and Nikkhoo [8] studied the capability of the piezoelectric patches acting as actuators to reduce the dynamic response of a thin rectangular plate under a moving mass traveling on an arbitrary trajectory. They employed a full-state classical linear optimal control algorithm with displacement–velocity feedback and employed the eigenfunction expansion method to solve the constitutive equation of motion. Their results were indicatory of a very good performance of the control system, especially for the loadings near the resonant states. Qiu et al. [9] explored the controllability of a clamped thin plate using piezoelectric patches as sensors and actuators analytically and experimentally. Their control algorithm was a combination of positive position feedback and proportional-derivative. Optimum locations of the piezoelectric sensors and actuators were also specified according to the piezoelectric control equation. They reported the feasibility of the utilized control algorithm and the efficiency of the optimal placement method. Huang and Hung [10] examined the vibration suppression of a simply supported plate under a harmonic force at its center by utilizing an active dynamic absorber. The absorber consists of two piezo patches bonded to the top and bottom surfaces of the plate and proper electric circuits. One patch is acting as the sensor to sense the velocity information while the other one acts as an actuator. Their results revealed this active absorber could effectively reduce the plate vibration for both controlled modes and the uncontrolled resonances.

On the other hand, one of the main concerns of structural engineers in designing bridge structures is to decrease the imposed vibrations by the moving loads during the life time of the structure. Moreover, if the inertial effects of the moving load are taken into account, the vibration amplitude of the bridge structure simulated as a single- or multi-span beam, increases appreciably for high moving mass weights and velocities [11–18]. Recently, Ouyang [19] has presented a valuable tutorial on different aspects of solids dynamics under moving loads. This review work introduces a variety of engineering problems in such a field of study and reflects the immense interest by the researchers in this subject area. Therefore, with the trend of light-weight bridge construction serving vehicles traveling at high speeds, the necessity of implementing a control system which could effectively reduce the dynamic response of these structures to any required values has become more crucial.

In this paper, the differential equation governing the vibration of single-span and multi-span thin beams having a number of piezoelectric actuators attached to their surfaces, is derived. Then, a linear classical optimal control algorithm with displacement–velocity and velocity–acceleration feedbacks is employed. Three types of external excitations, including a simple rectangular impulse, a moving load and a moving mass are considered. The method of eigenfunction expansion is used to solve the motion equation. Various numerical examples are presented to detect the performance of the control system on vibration suppression of the assumed structures even under severe dynamic loadings. The obtained results show the very good performance of the control system, specifically for the moving mass loading, where the span number increases.

2. Problem formulation

2.1. Single-span beams

A uniform continuous single-span Euler–Bernoulli beam with arbitrary boundary conditions is considered under any external excitation of $f(x,t)$. b and p subscripts signify the beam parameters and those related to the piezoelectric materials, respectively. E_bI_b is the flexural rigidity of the beam and ρ_bA_b is its mass per unit length. $z(x,t)$ stands for the deflection of the beam for any spatial location of x and time t. The origin of the x-axis matches the beam's left support (Fig. 1). Initial conditions are as $z(x,0)=g_1(x)$ and $\partial z(x,0)/\partial x=g_2(x)$, in which, $g_1(x)$ and $g_2(x)$ are any arbitrary continuous functions and the beam is supposed to be undamped. Moreover, n pairs of piezoelectric patches bonded symmetrically to the lower and upper surfaces of the beam, produce the required control forces, where the patches' nodes are located at x_{2i-1} and x_{2i} and $i=1,2,\ldots,n$. According to Fig. 1, $x_{2i}-x_{2i-1}=(l_p)_i$, in which, $(l_p)_i$ is the length of the ith piezoelectric actuator. Constitutive equation of one-dimensional piezoelectric patches is as follows [20]:

$$\begin{bmatrix} \sigma_{11} \\ E_3 \end{bmatrix} = \begin{bmatrix} E_p^E & -h_{31} \\ -h_{31} & \beta_{33}^T \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ D_3 \end{bmatrix} \tag{1}$$

In the above equation, E_p^E is Young's modulus of the piezoelectric material, h_{31} is the piezoelectric constant, β_{33}^T is the dielectric constant. Moreover, E_3 and D_3 are the out-of-plane components of the electrical field and electrical displacement, respectively.

The kinetic energy of the beam-piezoelectric patches system would be:

$$T=T_b+T_p, \tag{2}$$