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A model-based observer for state and stress estimation in structural and mechanical systems: Experimental validation

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ABSTRACT

In this paper we present the results from a validation study of a recently proposed model-based state observer for structural and mechanical systems. The observer uses a finite element model of the structure and noise contaminated measurements to estimate the state and stress time histories at arbitrary locations in the structure of interest. The initial conditions and unknown excitations are described by random vectors and random processes with known covariance and power spectral density. A laboratory model consisting of an aluminum cantilever beam was used to perform the experiment. Two types of loading conditions were tested: an impact hammer test and a band limited excitation delivered through a shaker. The results obtained with the proposed observer are compared to the measured stress at the locations of interest, and to estimates obtained using well-established estimation methods such as Luenberger observers and the Kalman filter. The main finding is that for all experiments conducted the proposed model-based observer yielded estimates with higher or comparable accuracy to all other methods considered, with the advantage of requiring significantly less computational effort and with a more direct and transparent implementation.

1. Introduction

Estimation of the stress and strain tensors at unmeasurable points in a solid or structure is a common problem encountered in many fields of science and engineering. Applications range from fatigue monitoring of structural systems [19,10], estimation of the force through a joint in the human body [12], to estimating heart kinematics [14]. In fatigue analysis, for example, the estimated stress time histories can be used as inputs to damage functions and consequently be used to estimate the failure risk due to oscillatory loads [3]. If global system response such as accelerations at a number of locations is measured, the objective is then to reconstruct unmeasured quantities of interest (QoI) from the available measurements and a model of the system.

State estimation methods typically fall under two categories: data interpolation methods [23,13,5], and methods based on control theory that explicitly account for the dynamic relationship between the measurements, measurement noise, disturbances and the QoI [21]. Traditionally, data interpolation methods provide acceptable results for situations where the QoI are governed by a number of vibration modes that is less than the number of measurements and when the analyst knows a priori which modes are dominant. Otherwise, interpolation methods typically fail to provide accurate results [2].

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An alternative approach is a family of methods based on estimation and control theory, known as observers [15]. An observer is a dynamical system which serves as an estimator for the state of the system of interest, and is formulated as a feedback loop driven by a linear combination of the difference between the system output measurements and the corresponding observer response. An observer is defined as convergent if the observer's state and the system's state converge with time, i.e., the error is guaranteed to converge to zero asymptotically.

In observer design two cases are typically encountered: an estimate of the state is sought due to unknown initial conditions and(or) unmeasured excitations. In the case of unknown initial conditions and a continuous time system description, a convergent observer is formulated by means of shifting the poles of the observer, with respect to those of the system, towards the left of the complex plane. These types of observers are known as Luenberger observers [15]. In the case of unmeasured excitations, one of the most celebrated state estimation algorithms is the Kalman filter (KF), a recursive Bayesian estimation algorithm which allows for optimal estimation (in the Euclidean sense) of the state trajectory of a linear system on the basis of a state-space model of the system and noise-contaminated measurements [11]. The two fundamental assumptions in the KF are as follows: (i) the system is linear and (ii) excitations and measurement noise are realizations of Gaussian random processes. In most applications, only systems with a limited number of degrees of freedom are considered, mainly due to the computational difficulties that arise when implementing the KF on high dimensional systems [18].

In addition to the computational issues, theoretical issues have also been raised related to the consistency of first order observers when applied to second order symmetric systems; especially loss of symmetry, definiteness and the fact that first order observers might yield an estimate of the state that does not correspond to the physical state sought, i.e., it does not conserve the internal consistency between certain variables, such as the fact that the estimated velocities must be derivatives of the estimated displacements. This was demonstrated by Balas [1] and Hernandez [8], who concluded that unless certain restrictions are placed on the observer formulation these inconsistencies will occur and can potentially lead to significant estimation errors. Observers that satisfy the consistency requirement are known as natural observers. It has been shown that the KF is not a natural observer for linear symmetric second order models typically used in structural dynamics [8].

In this paper we report the results of various experiments conducted by the authors to validate a natural finite element model based observer developed by Hernandez [8]. The objective of the experiments is to estimate the stress time histories at arbitrary locations in a cantilever beam using noise contaminated acceleration response measurements. The proposed observer resembles the KF, but with the capability of direct implementation as a modified finite element model of the system of interest and with significantly reduced computational effort. To the best knowledge of the authors, this constitutes the first published experimental validation study regarding the use of observers to estimate stress and strain fields in structures. Previous published work on the use of observers in structural dynamics investigated their applicability for estimating acceleration at unmeasured locations [22].

The paper is laid out as follows; a brief introduction to the theory behind the proposed model-based observer (MBO) is presented in the first section. Additionally, some essential aspects of observer theory and Kalman filtering are discussed. This is followed by a section describing the laboratory experiments and the results. It is shown that the estimates from the proposed model-based observer outperform the Luenberger observer and are comparable (and sometimes superior) to the Kalman filter estimates.

2. Theoretical background

In this paper we restrict our attention to systems whose dynamic response can be accurately described by the following matrix ordinary differential equation:

$$\mathbf{M}\ddot{q}(t)+\mathbf{C}_d\dot{q}(t)+\mathbf{K}q(t)=\mathbf{b}_1u(t)+\mathbf{b}_2w(t) \tag{1}$$

where $\mathbf{M}=\mathbf{M}^T>0\in\mathbb{R}^{N\times N}$ is the mass matrix, $\mathbf{C}_d=\mathbf{C}_d^T>0\in\mathbb{R}^{N\times N}$ is the damping matrix and $\mathbf{K}=\mathbf{K}^T>0\in\mathbb{R}^{N\times N}$ is the stiffness matrix. The vector $q(t)\in\mathbb{R}^{N\times 1}$ is the displacement vector of the N degrees of freedom, $\mathbf{b}_1\in\mathbb{R}^{N\times r}$ defines the spatial distribution of the excitation $u(t)\in\mathbb{R}^{r\times 1}$ and $\mathbf{b}_2\in\mathbb{R}^{N\times p}$ defines the spatial distribution of the unmeasured excitation $w(t)\in\mathbb{R}^{p\times 1}$. By defining the state vector as $x(t)=[q^T(t)\ \dot{q}^T(t)]^T$, Eq. (1) can be written in first order form as

$$\dot{x}(t)=\mathbf{A}_cx(t)+\mathbf{B}_1u(t)+\mathbf{B}_2w(t) \tag{2}$$

where the matrices $\mathbf{A}_c$, $\mathbf{B}_1$ and $\mathbf{B}_2$ are defined as

$$\mathbf{A}_c=\begin{bmatrix}\mathbf{0}_{N\times N} & \mathbf{I}_{N\times N}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}_D\end{bmatrix} \tag{3}$$

$$\mathbf{B}_1=\begin{bmatrix}\mathbf{0}_{N\times r}\\ \mathbf{M}^{-1}\mathbf{b}_1\end{bmatrix},\quad \mathbf{B}_2=\begin{bmatrix}\mathbf{0}_{N\times p}\\ \mathbf{M}^{-1}\mathbf{b}_2\end{bmatrix} \tag{4}$$

Feedback measurements $y(t)$ at discrete points in the structure are given by

$$y(t)=\mathbf{C}x(t)+\mathbf{D}_1u(t)+\mathbf{D}_2w(t)+\nu(t) \tag{5}$$

where for acceleration feedback $\mathbf{C}=\mathbf{c}_2[-\mathbf{M}^{-1}\mathbf{K}\ -\mathbf{M}^{-1}\mathbf{C}_d]$, $\mathbf{D}_1=\mathbf{c}_2\mathbf{M}^{-1}\mathbf{b}_1$, $\mathbf{D}_2=\mathbf{c}_2\mathbf{M}^{-1}\mathbf{b}_2$; for displacement feedback $\mathbf{C}=[\mathbf{c}_2\ \mathbf{0}_{m\times N}]$ and for velocity feedback $\mathbf{C}=[\mathbf{0}_{m\times N}\ \mathbf{c}_2]$. For displacement and velocity measurements $\mathbf{D}_1$ and $\mathbf{D}_2$ are zero