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Subspace-based identification of a nonlinear spacecraft in the time and frequency domains

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ABSTRACT

The objective of the present paper is to address the identification of a strongly nonlinear satellite structure. To this end, two nonlinear subspace identification methods formulated in the time and frequency domains are exploited, referred to as the TNSI and FNSI methods, respectively. The modal parameters of the underlying linear structure and the coefficients of the nonlinearities will be estimated by these two approaches based on periodic random measurements. Their respective merits will also be discussed in terms of both accuracy and computational efficiency and the use of stabilisation diagrams in nonlinear system identification will be introduced. The application of interest is the SmallSat spacecraft developed by EADS-Astrium, which possesses an impact-type nonlinear device consisting of eight mechanical stops limiting the motion of an inertia wheel mounted on an elastomeric interface. This application is challenging for several reasons including the non-smooth nature of the nonlinearities, high modal density and high nonproportional damping.

1. Introduction

Subspace methods are commonly recognised as powerful identification tools for linear and time-invariant systems, arguably because they are non-iterative and hence computationally tractable, and naturally applicable to multi-input multi-output systems. They were first introduced in the time domain [1,2] and later revisited to consider frequency response functions [3,4] and power spectra [5] as input data. Since then, subspace-based algorithms have been successfully applied to a wide variety of real-life systems, *e.g.* in control [6] or biomedical [7] engineering. In the field of structural dynamics, they are routinely used not only for experimental and operational modal analysis [8], but also for advanced processing such as damage detection and structural health monitoring [9].

However, because engineering structures are known to be prone to nonlinearity [10], there is a crucial need for extending linear subspace methods to a practical nonlinear analogue. In this context, the first contribution is due to Lacy and Bernstein [11], who derived a time-domain algorithm applicable to nonlinear mechanical systems. A numerically robust implementation of this algorithm, referred to as the time-domain nonlinear subspace identification (TNSI) method, was then proposed in [12], yielding superior accuracy. More recently, a dual approach has been developed in the frequency domain, termed

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frequency-domain nonlinear subspace identification (FNSI) method [13], which allows the computational burden to be reduced.

This time–frequency duality is a frequent occurrence in the technical literature. Specifically, the restoring force surface (RFS) [14] and the nonlinear identification through feedback of the outputs (NIFO) [15] methods can be seen as two direct least-squares estimators in the time and frequency domains, respectively. One should also cite the orthogonalised reverse path (ORP) [16] and the conditioned reverse path (CRP) [17] methods that are manifestly two other dual techniques. There were some comparative studies reported in the past few years, in particular between the RFS and CRP methods in [18,19] or between the RFS and NIFO methods in [20,21].

The objective of the present paper is to address the identification of a strongly nonlinear satellite structure, using the TNSI and FNSI methods. The modal parameters of the underlying linear structure and the coefficients of the nonlinearities will be estimated by these two approaches based on periodic random measurements. Their respective merits will also be discussed in terms of both accuracy and computational efficiency and the use of stabilisation diagrams in nonlinear system identification will be introduced. The application of interest is the SmallSat spacecraft developed by EADS-Astrium, which possesses an impact-type nonlinear device consisting of eight mechanical stops limiting the motion of an inertia wheel mounted on an elastomeric interface. This application is challenging for several reasons including the non-smooth nature of the nonlinearities, high modal density and high non-proportional damping.

The paper is organised as follows. Section 2 introduces the theoretical background of the TNSI and FNSI methods. The SmallSat spacecraft is then described in Section 3 and details about its finite element modelling are given. Section 4 next addresses the identification of the SmallSat structure based on two synthetic data sets corrupted by additive noise. In the first numerical experiment, the excitation is chosen such that a single nonlinearity is activated and its identification is discussed via two sets of processed channels. The use of stabilisation diagrams in nonlinear system identification is introduced and the estimation of the clearance beyond which impacts occur in the nonlinear connection is also carried out. A second identification case is then analysed involving the strong activation of eight nonlinearities. Finally, the random nature of the identification results due to the finite length of the data sets and the presence of measurement noise is carefully analysed through repeated experiments. The conclusions of the study are summarised in Section 5.

2. Subspace identification of nonlinear mechanical systems in the time and frequency domains

As described in the review paper [10], the identification of nonlinear mechanical systems can be seen as a progression through nonlinearity detection, characterisation and parameter estimation. Following the detection step whose goal is obvious, characterisation is concerned with nonlinearity localisation and model selection. The model parameters are then estimated, for example, by means of least-squares fitting or nonlinear optimisation depending upon the method considered. The common aim of the TNSI and FNSI methods is to address this latter step and so to estimate nonlinear stiffness and damping coefficients together with the frequency response function (FRF) matrix of the underlying linear system.

2.1. State-space model and problem statement

The vibrations of nonlinear systems are governed by the time-continuous model:

$$M\ddot{q}(t)+C_v\dot{q}(t)+Kq(t)+f(q(t),\dot{q}(t))=p(t) \tag{1}$$

where M, C_v, and $K \in \mathbb{R}^{r\times r}$ are the linear mass, viscous damping, and stiffness matrices, respectively; $q(t)$ and $p(t) \in \mathbb{R}^r$ are the generalised displacement and external force vectors, respectively; $f(t) \in \mathbb{R}^r$ is the nonlinear restoring force vector, and r is the number of degrees of freedom (DOFs) of the structure obtained after spatial discretisation. The amplitude, direction, location and frequency content of the excitation $p(t)$ determine in which regime the structure behaves. As in Ref. [15], the joint effect of the s lumped nonlinearities in the system is modelled using a summation of the form:

$$f(q(t),\dot{q}(t))=\sum_{j=1}^{s}\eta_j g_j(q(t),\dot{q}(t))b_j. \tag{2}$$

Each term contains an unknown nonlinear coefficient η_j and the corresponding functional form $g_j(t)$, which is assumed to be known. Nonlinearity localisation is specified using a vector of Boolean values, $b_j \in \mathbb{R}^r$. In the technical literature about subspace methods, first-order state-space models are preferred to the second-order description of the dynamics in Eq. (1), because of the intrinsic capability of a state-space model to encompass multi-input multi-output (MIMO) systems. Considering that displacements are measured and defining the state vector $x=(q^{\mathrm{T}}\ \dot{q}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^n$, the equations of motion are recast into

$$\begin{cases}\dot{x}(t)=A_c x(t)+B_c^{nl}g(q(t),\dot{q}(t))+B_c p(t)\\ q(t)=C_c x(t)+D_c p(t)\end{cases} \tag{3}$$

where subscript c stands for *continuous-time*; $A_c \in \mathbb{R}^{n\times n}$, $B_c^{nl} \in \mathbb{R}^{n\times s}$, $B_c \in \mathbb{R}^{n\times r}$, $C_c \in \mathbb{R}^{r\times n}$ and $D_c \in \mathbb{R}^{r\times r}$ are the state, nonlinear coefficient, input, output and direct feed-through matrices, respectively; $g(t) \in \mathbb{R}^s$ gathers the basis functions $g_j(t)$, and