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Design sensitivity and Hessian matrix of generalized eigenproblems

Li Li, Yujin Hu*, Xuelin Wang

School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, PR China

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ABSTRACT

A generalized eigenproblem is formed and its normalizations are presented and discussed. Then a unified consideration of the computation of the sensitivity and Hessian matrix is studied for both the self-adjoint and non-self-adjoint cases. In the self-adjoint case, a direct algebraic method is presented to determine the eigensolution derivatives simultaneously by solving a linear system with a symmetric coefficient matrix. In the non-self-adjoint case, an algebraic method is presented to determine the eigensolution derivatives directly and simultaneously without having to use the left eigenvectors. In this sense, the method has advantages in computational cost and storage capacity. It is shown that the second order derivatives of eigensolutions can also be obtained by solving a linear system and the computational effort of obtaining Hessian matrix is reduced remarkably since only the recalculation of the right-hand vector of the linear system is required. The presented methods are accurate, compact, numerically stable and easy to implement. Finally, two transcendental eigenproblem examples are used to demonstrate the validity of the presented methods. The first example is considered as an example of the case of non-self-adjoint systems, which can result from feedback control systems. The other example is used to illustrate the case of self-adjoint systems by considering the three bar truss structure which is a viscoelastic composite structure and consists of two aluminum truss components and one viscoelastic truss. In addition, the capacity of predicting the changes of eigenvalues and eigenvectors with respect to the changes of design parameters is studied.

1. Introduction

Design sensitivity analysis of mechanical and engineering structures, often referred to as design derivatives, deals with the calculation of the rate of performance measures from changes in the design variables. A significant body of work has been carried out on the computation and the application of design sensitivity. (For details, [1, 2, 3, 4, 5] or [6].). A detailed discussion of developments related to the design sensitivity of the eigenvalue problem may refer to Adelman and Haftka [3], Haftka et al. [4] or Chen [6]. Eigensensitivity analysis has become an integral part of many engineering design methodologies such as model updating [7,8], approximate reanalysis technique [9], structural design optimization [10,11], structural health monitoring [12], operational mode shape normalization [13], eigensolution calculation [14], structural or systemic reliability [15], dynamic modification [16], and random eigenvalue problem [17]. As pointed by Mottershead et al. [7], the eigensensitivity-based method is probably the most successful of the many methods to the problem of model updating. Although eigenvalue sensitivity can be

* Corresponding author. Tel.: +86 27 87543972.
E-mail addresses: lili_em@126.com, lili_em@hust.edu.cn (L. Li), yjhu@mail.hust.edu.cn (Y. Hu).

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obtained in a straightforward way, determining eigenvector derivatives raises several challenges (e.g., *the singular issue*). Several methods have been developed for the calculation of eigensensitivity for the past decades.

In the case of undamped systems, the equations of motion of the free vibration can be given by

$$(s\mathbf{M}-\mathbf{K})\mathbf{u}(s)=\mathbf{0} \tag{1}$$

Here $\mathbf{K}$ and $\mathbf{M}\in\mathbb{R}^{N\times N}$ are, respectively, the stiffness and mass matrices, where N is the system size and $\mathbb{R}$ denotes the space of real numbers. The eigenvalues λ_i are the roots of the characteristic equation, $\det|\mathbf{K}-s\mathbf{M}|=0$. The eigenvalues $\lambda_i=\omega_i^2$ where ω_i is the ith undamped circular frequency. Fox and Kapoor [18] first derived the sensitivity expressions of the corresponding eigenproblem in terms of all the mode shapes (often this method is called the modal method). Plaut and Huseyin [19] studied the modal method for general asymmetric or non-self-adjoint systems. The undamped asymmetric systems may arise by premultiplying Eq. (1) using the inverse of mass matrix. Also, it exists in some physical problems, e.g., in the case of undamped circulatory systems, one has (see [20] or [21]):

$$(s\mathbf{M}-\mathbf{K}-\mathbf{H})\mathbf{u}(s)=\mathbf{0} \tag{2}$$

Here $\mathbf{H}$ denotes circulatory matrix, and is a skew-symmetric matrix, i.e., $\mathbf{H}=-\mathbf{H}^T$ where superscript T is the matrix transpose. The previous equation can be considered as an asymmetric stiffness system. Some physical systems may also have asymmetric mass matrices which may arise from some given inertial forces (see, e.g., [22]). Often only a few lower modes are considered in the dynamic analysis of large-scale engineering problem, it means that approximated eigensensitivity is generally evaluated and the mode truncation error is therefore introduced. The corrections to the modal method have been investigated by many authors, e.g., Wang [23], Moon et al. [24], Chen [6]; and Liu and Chen [25]. The modal method is a powerful one and extensively applied in engineering (see, e.g. in model updating problem [7,26,27] or direct and inverse eigenproblems [11,16,28,29]). To reduce the number of mode shapes involved, Nelson [30] presented a technique to calculate each eigensolution sensitivity only in terms of the eigensolution of interest. Friswell [31] extended Nelson's method to compute the second and higher order eigensolution derivatives. Sutter et al. [32] pointed out Nelson's method is more efficient than the modal method due to the fact that the modal method needs all or most of eigenvectors to find each eigenvector derivative. Fox and Kapoor [18] also suggested a direct algebraic method to calculate the eigensensitivities of symmetric undamped systems. Later, Garg [33], Rudisill [34], Rudisill and Chu [35] investigated the algebraic method of general asymmetric eigensystems. In 1997, Lee and Jung [36,37] presented an algebraic method, which calculates the eigenderivatives by solving a linear system with a symmetric coefficient matrix for symmetric undamped systems with distinct and repeated eigenvalues, respectively. The algebraic method is also a powerful one and extensively applied in engineering [38–40] since it is accurate, compact, and numerically stable. Several other methods have been developed for the calculation of eigensensitivity such as the finite difference method [41–43], the substructuring method [44–46] and iterative method [47–50]. Murthy and Haftka [51] surveyed the methods of computing the eigensensitivity of a general complex matrix. Repeated eigenvalues or nearly equal eigenvalues may exist in typical structural or mechanical systems for certain reasons. Juang et al. [52] gave a proof of the existence of the eigensensitivity for non-defective systems with repeated eigenvalues. Many studies have been devoted to the calculation of the sensitivities of eigensystems with repeated eigenvalues (see Refs. [6,53–67] for details).

Increasing the use of damping technology (including composite structural materials, active control and damage tolerant systems) in rockets, spacecrafts, satellites, ships and automobiles [68,69], the need to consider the sensitivity of damped eigensystems is more than ever before. Eigensensitivity analysis of damped systems has therefore received much attention over the past two decades. The equations of motion of the free vibration for a linear nonviscously (viscoelastically) damped system can be expressed by [70–75]

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\int_0^t \mathbf{g}(t-\tau)\dot{\mathbf{q}}(\tau)d\tau+\mathbf{K}\mathbf{q}(t)=\mathbf{0} \tag{3}$$

Here $\mathbf{g}(t)$is the matrix of kernel functions and $t\in\mathbb{R}^+$ denotes time. Taking the Laplace transform of the previous equation gives

$$(s^2\mathbf{M}+s\mathbf{G}(s)+\mathbf{K})\mathbf{u}(s)=\mathbf{0} \tag{4}$$

where $\mathbf{u}(s)=L[\mathbf{q}(t)]$, $\mathbf{G}(s)=L[\mathbf{g}(t)]$ and $L[]$denotes the Laplace transform. The dynamic equations of motion similar to Eqs. (3) and (4) may arise in many different subjects, such as viscoelastic structures [14], dynamic of railway track [76], ship dynamics [77], energy dissipation in structural joints [78], damping mechanism in composite beams [79], vibration isolation [80,81] or control problems [82]. Many researchers presented different representations of the kernel functions during the last decades (for details, refer to [74,75]). In the special case, when $\mathbf{g}(t-\tau)=\mathbf{C}\delta(t-\tau)$ where $\mathbf{C}$ is a viscous damping matrix and $\delta(t-\tau)$ is the Dirac delta function, Eq. (4) is reduced to

$$(s^2\mathbf{M}+s\mathbf{C}+\mathbf{K})\mathbf{u}(s)=\mathbf{0} \tag{5}$$

which is a familiar viscously damped system [83]. Hence the nonviscous damping model is considered as a further generalization of the familiar viscous damping. Damped asymmetric systems may arise in control systems or physical systems (e.g., rotor dynamic systems). The equations of motion of the free vibration for rotor dynamic systems can be given by

$$[s^2\mathbf{M}+s(\mathbf{G}(s)+\mathbf{G}_y)+\mathbf{K}+\mathbf{H}]\mathbf{u}(s)=\mathbf{0} \tag{6}$$