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Iterative finite element model updating in the time domain

Eric M. Hernandez [a,*], Dionisio Bernal [b]

[a] School of Engineering, College of Engineering and Mathematical Sciences, University of Vermont, 33 Colchester Ave., 301 Votey Bldg. Burlington, VT 05405, USA
[b] Department of Civil and Environmental Engineering, Center for Digital Signal Processing, Northeastern University, Boston, MA 02115, USA

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ABSTRACT

An iterative time domain formulation for finite element model updating in structural dynamics is presented. The approach is supported on a derivation showing that the discrepancy between observations and model predictions can be expressed as a convolution between the state of the system and a sequence of pseudo-Markov parameters which are linear in the change of the free parameters. The approach is illustrated by updating all the stiffness and damping parameters of a twenty degree of freedom shear beam using four noise contaminated measurements.

1. Introduction

Finite Element (FE) model updating is the process by which an initial model (referred to in this paper as a *nominal model*) of a dynamical system is modified (updated) in order to minimize a metric of the difference between system response measurements and model predictions [2]. In the existing literature, two general approaches can be distinguished; those that reduce the measured data to spectral features by means of system identification and those that operate directly on the data without reduction. Methods that operate on the basis of spectral reduction update model parameters to reduce the difference between the identified and the updated FE model eigenvalues and eigenvectors. Among these, sensitivity based methods are used most commonly [7]. Well-known difficulties when applying sensitivity methods are determining correct pairing between the FE model and the identified mode shapes, limitations on the number of modes and frequencies that can be accurately extracted from the measurements and the hypersensitivity of identification algorithms to computational user-defined parameters, especially in mode shape identification.

Methods that operate directly on the measurements, especially in cases where the excitation is not fully characterized deterministically, are typically implemented using adaptive data assimilation methods such as extended Kalman filters [4] and, most recently, unscented Kalman filters [9,10]. The aim is to minimize the 2-norm of the difference between measurements and estimator output time histories. Adaptive data assimilation methods operate by expanding the state vector to include model parameters and use sequential Bayesian estimation to estimate the expanded state as measurements become available, thus resulting in a nonlinear estimation problem.

The updating approach presented in this paper rests on a derivation showing that the difference between measurements and predictions, i.e., the output residuals, can be expressed as a convolution between the system state and a set of pseudo-Markov

* Corresponding author. Tel.: +1 802 656 3334; fax: +1 802 656 3358.
E-mail addresses: eric.hernandez@uvm.edu (E.M. Hernandez), bernal@neu.edu (D. Bernal).

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parameters. In the proposed approach the updated FE model evolves from an initial estimate $M_0(\theta_o)$ to a model that at the jth iteration is denoted as $M_j(\theta_j)$ where θ are the free model parameters. In contrast with standard optimization techniques, the proposed method does not progress toward the solution using model sensitivities but rather takes advantage of the fact that given the correct state sequence the update of the discrete time transition matrix is a linear function of the output residuals. Since the true state of the system is not known, state estimation is required. State estimation is accomplished through an observer derived on the premise that the main source of output discrepancy lies in model error [3]. In essence, the main contribution of the paper consists in decoupling the nonlinear estimation problem typically encountered in the non-linear Kalman filter approach into two sequential linear problems, namely a linear state estimation and a linear least square problem.

To make the update linear in each cycle the transition and input to state matrices in discrete time are taken as first order approximations, which can be made as accurate as required by using a small enough discretization time step. The two operating assumptions are that the mass matrix is known and that the measurements are part of the state vector. This paper describes the approach and contains a numerical section where the method is implemented to update all the stiffness and damping coefficients of a 20-DOF shear beam using noise-contaminated measurements from four coordinates.

2. State estimation

The most popular methodology for recursive state estimation and tracking is the Kalman filter (KF) [6]. The KF performs optimally in linear systems whenever discrepancies between observations and model predictions arise from process and measurement noises of known covariance and white spectra [5]. In the situation that concerns us here, however, the operating assumptions differ from those in the derivation of the Kalman filter and an alternative method is desirable. In particular, in our case the bulk of the discrepancy between model predictions and measurements derive from inaccuracy in the model itself. Examination of the theory of Robust Kalman filtering [8], which extends the standard Kalman filter to the situation where the model is uncertain, showed that the methodology is unlikely to offer a practical solution for large dimensional structural problems. In particular, the methodology involves the solution of two Ricatti equations, demands a specific parameterization of the errors in the state transition and input to state matrices and, as in the standard Kalman filter, requires specification of covariance matrices for the process and the measurement noise. A simple estimator designed explicitly to consider errors in stiffness and damping that does not require parameterization of the model error appears in Hernandez and Bernal [3] and is used here in the state estimation. The recursive state estimator is given as

$$\hat{x}_{k+1} = (\mathbf{I}-\mathbf{C}^T\mathbf{C})\hat{\mathbf{A}}\hat{x}_k + (\mathbf{I}-\mathbf{C}^T\mathbf{C})\hat{\mathbf{B}}u_k + \mathbf{C}^T y_{k+1} \tag{1}$$

where unless stated otherwise $\mathbf{I}$ is the identity matrix of dimension N, $\hat{\mathbf{A}} \in R^{N\times N}$ and $\hat{\mathbf{B}} \in R^{N\times r}$are the state transition and input-to-state matrices of the nominal model respectively. $\mathbf{C} \in R^{m\times N}$ is the state-to-output matrix, $\hat{x}_{k+1} = R^{N\times 1}$is the estimated state at time $t=(k+1)\Delta t$, where Δt is the time step, $u_k \in R^{r\times 1}$ is the measured input at time $t=k\Delta t$ and $y_{k+1} \in R^{m\times 1}$ is the output vector of available measurements at time $t=(k+1)\Delta t$. Throughout the paper the notation $\hat{z}$ will be used to denote the estimated value of the quantity z.

The discrete time state transition matrix $\hat{\mathbf{A}}$ in Eq. (1) is related to the continuous time $\mathbf{A_c}$ by the following matrix exponential relation

$$\hat{\mathbf{A}} = e^{\hat{\mathbf{A}}_c\Delta t} = \mathbf{I} + \hat{\mathbf{A}}_c\Delta t + h.o.t. \tag{2}$$

For a second order linear structural system $\mathbf{A_c}$ is defined as

$$\hat{\mathbf{A}}_\mathbf{c} = \begin{bmatrix} \mathbf{0}_{q\times q} & \mathbf{I}_{q\times q} \\ -\mathbf{M}^{-1}\hat{\mathbf{K}} & -\mathbf{M}^{-1}\hat{\mathbf{C}}_\mathbf{D} \end{bmatrix} \tag{3}$$

where $\mathbf{M} = \mathbf{M}^T > 0 \in R^{q\times q}$ is the mass matrix of the system (assumed known), $\hat{\mathbf{C}}_D = \mathbf{C}_D^T \geq 0 \in R^{q\times q}$ is the damping matrix and $\hat{\mathbf{K}} = \hat{\mathbf{K}}^T \geq 0 \in R^{q\times q}$ is the stiffness matrix of the nominal model. The matrix $\hat{\mathbf{B}}$ depends on the input to state matrix in continuous time $\mathbf{B_c}$ and on the operating assumption regarding the behavior of the inputs. The $\mathbf{B_c}$ matrix is

$$\mathbf{B_c} = \begin{bmatrix} \mathbf{0}_{q\times r} \\ \mathbf{M}^{-1}b_2 \end{bmatrix} \tag{4}$$

where $b_2 \in R^{q\times r}$ is a vector that describes the spatial position of the input forces. On the premise that the input is a band limited function it is shown in Bernal [1] that

$$\mathbf{B} = \hat{\mathbf{A}}\mathbf{B}_c\Delta t \tag{5a}$$

Substituting Eq. (2) into Eq. 5a gives

$$\hat{\mathbf{B}} = (\mathbf{I} + \hat{\mathbf{A}}_c\Delta t + h.o.t.)\mathbf{B}_c\Delta t \approx \mathbf{B}_c\Delta t \tag{5b}$$

which shows that the first order approximation of the discrete input to state matrix is not a function of the stiffness or the damping and can thus be treated as known. The $\mathbf{C}$ matrix is known and defined by the output equation as

$$y_k = \mathbf{C}x_k + v_k \tag{6}$$

where ν is the output noise.