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Localization and quantification of damage in beam-like structures using sensitivities of principal component analysis results

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ABSTRACT

Principal component analysis (PCA) is known as an efficient method for dynamic system identification and diagnosis. This paper addresses a damage diagnosis method based on sensitivities of PCA in the frequency domain for linear-form structures. The aim is not only to detect the presence of damage, but also to localize and to evaluate it. The Frequency response functions measured at different locations on the beam are considered as data for the PCA process. Sensitivities of principal components obtained from PCA to beam parameters are computed and inspected according to the location of sensors; their variation from the healthy state to the damaged state indicates damage locations. The damage can be evaluated next providing that a structural model is available; this evaluation is based on a model updating procedure. It is worth noting that the diagnosis process does not require a modal identification achievement. Both numerical and experimental examples are used for better illustration.

1. Introduction

A structural damage may disturb or threaten the normal working conditions of a system. For that reason, questions of the detection, localization and severity estimation of those events have attracted the attention of countless engineering researchers in recent times. The detection, localization and assessment of the damage allow to guarantee safety as well as to reduce maintenance and repair costs.

The problem of damage localization and assessment has been approached from many directions in the last decade. Often based on monitoring modal features, these processes can be achieved by using an analytical model and/or promptly by measurement. Damage can cause change in structural parameters, involving the mass, damping and stiffness matrices of the structure. Thus many methods deal directly with these system matrices. The finite element method (FEM) is an efficient tool in this process [1]. The problem of detection may be resolved by this method through model updating or sensitivity analysis. For damage localization and evaluation, model updating is utilized to reconstruct the stiffness perturbation matrix [2]. This may be combined with a genetic algorithm [3] or based on modal parameter sensitivity [4]. Those approaches require a well fitted numerical model to compare with the actual system.

Measurement is also widely used for damage detection because of their availability in practice. Yan and Golinval [5] achieved damage localization by analyzing flexibility and stiffness without system matrices, using time data measurements. Koo et al. [6] detected and localized low-level damage in beam-like structures using deflections obtained

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by modal flexibility matrices. Following localization, Kim and Stubbs [7] estimated damage severity based on the mode shape of a beam structure. Rucka and Wilde [8] decomposed measured FRFs by continuous wavelet transform in order to achieve damage localization. Based also on continuous wavelet transform (CWT), Bayissa et al. [9] analyze measured time responses to extract principal structural response features. The authors have used the zeroth-order moment (ZOM) to detect and localize damage in a plate model and a full-scale bridge structure. Cao and Qiao [10] recently used a novel Laplacian scheme for damage localization. Other authors have located damage by comparing identified mode shapes [11] or their second-order derivatives [12] in varying levels of damage. Sampaio et al. [13] extended the method proposed in [12] through the use of measured FRFs.

It can be seen that natural frequency sensitivity has also been used extensively for the purposes of damage localization. Ray and Tian [11] discussed the sensitivity of natural frequencies with respect to the location of local damage. In that study, damage localization involved the consideration of mode shape change. Other authors [14–16] have located damage by measuring natural frequency changes both before and after the occurrence of damage. However, such methods, based on frequency sensitivity with respect to damage variables require an accurate analytical model. Jiang and Wang [17] extended the frequency sensitivity approach by eliminating that requirement. However, an optimization scheme is still needed to estimate the unknown system matrices through an identified model using input–output measurement data.

It is not only the issue of localization that has become the subject of recent study; the assessment of damage is also increasingly attracting the interest of researchers [2–4,7,15]. Yang et al. [18] estimated damage severity by computing the current stiffness of each element. They used Hilbert–Huang spectral analysis based only on acceleration measurements using a known mass matrix assumption. Taking an alternative approach, other authors have used methods involving the updating of a finite element model of the examined structure and have used sensitivity analysis to discover the effective parameter. For example, Messina et al. [14] estimated the size of defects in a structure based on the sensitivity of frequencies with respect to damage locations where all the structural elements were considered as potentially damaged sites. Teughels and De Roeck [19] identified damage in the highway bridge. They updated both Young's modulus and the shear modulus using an iterative sensitivity based FE model updating method. The damage in a 50-year old bridge was identified by Reynders et al. [20], where eigenfrequencies, mode shape vectors and modal curvature vectors were used for the model updating. In Ref. [21,22], the damage localization and quantification were achieved in reinforced concrete frames by comparing eigenfrequencies and mode shapes with different optimization techniques.

The sensitivity analysis is used in this study for resolving the problems of damage localization and evaluation. Natural frequencies are known to be successful in characterizing dynamical systems. Mode shapes are used in model updating because they give information regarding the spatial distribution of damage whereas it is not possible to locate the damage by using the frequencies only in the objective function constructed for minimization. Hence, we use not only sensitivity of frequency, but also of mode shape in this work. A modal identification is not necessary for the objective of localization. In monitoring the distortion of a sensitivity vector, the localization may be carried out in the first step. An analytical model is then needed for model updating, and this enables the assessment of the damage.

2. Sensitivity analysis based on principal component analysis

The dynamical responses are conditional on many factors including the parameters related to material, geometry and dimensions. We know that the dynamic behavior of a system is fully characterized by its modal parameters. Sensitivity analysis of modal parameters may be a useful tool for uncovering and locating damaged or changed components of a structure. Principal component analysis (PCA) of the response matrix of the system provides a way to extract modal features (i.e. principal directions). This approach is used in this study to examine modal parameter sensitivities.

Given the observation matrix $\mathbf{X}^{m\times N}$ which contains the dynamic responses of the system where m is the number of measured co-ordinates and N is the number of time instants. We will assume that it depends on a vector of parameters $\mathbf{p}$. The observation matrix $\mathbf{X}$ can be decomposed using singular value decomposition (SVD):

$$\mathbf{X} = \mathbf{X}(\mathbf{p}) = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \tag{1}$$

where $\mathbf{U}$ and $\mathbf{V}$ are two orthogonal matrices, whose columns represent respectively left and right singular vectors; $\boldsymbol{\Sigma}$ contains singular values of descending importance: $\sigma_1 > \sigma_2 > \ldots > \sigma_m$.

The sensitivity of a quantity to a parameter is described by the first and higher orders of its partial derivatives with respect to the parameter. A sensitivity analysis is performed here by taking the derivative of the observation matrix with respect to $\mathbf{p}$:

$$\frac{\partial \mathbf{X}}{\partial \mathbf{p}} = \frac{\partial \mathbf{U}}{\partial \mathbf{p}}\boldsymbol{\Sigma}\mathbf{V}^T + \mathbf{U}\frac{\partial \boldsymbol{\Sigma}}{\partial \mathbf{p}}\mathbf{V}^T + \mathbf{U}\boldsymbol{\Sigma}\frac{\partial \mathbf{V}^T}{\partial \mathbf{p}} \tag{2}$$

This shows that the sensitivity of the system dynamic response depends on the sensitivity of each SVD term. Junkins and Kim [23] developed a method to compute the partial derivatives of SVD factors. The singular value sensitivity and the left and right singular vector sensitivity are simply given by the following equations:

$$\frac{\partial \sigma_i}{\partial p_k} = \mathbf{U}_i^T \frac{\partial \mathbf{X}}{\partial p_k}\mathbf{V}_i; \quad \frac{\partial \mathbf{U}_i}{\partial p_k} = \sum_{j=1}^{m} \alpha_{ji}^k \mathbf{U}_j; \quad \frac{\partial \mathbf{V}_i}{\partial p_k} = \sum_{j=1}^{m} \beta_{ji}^k \mathbf{V}_j \tag{3}$$