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Modified Quadratic Compression Method for mass and stiffness updating

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ABSTRACT

The paper presents the modified Quadratic Compression Method (QCM) for both mass and stiffness model updating. The modeling error is defined in a parametric setup, i.e. with pre-specified principal submatrices multiplied by unknown scalar parameters. The optimal parameters are obtained by minimizing the error in a squared down version of the eigenvalue equation, and of the mass orthogonality condition, thus with reduced computation yet with no loss of information. The method has closed ties with Minimization of the Error in the Constitutive Equation (MECE), and in some cases is shown to belong to that class with a particular choice of the weighting matrix. Theoretical analysis of the propagation of the noise into the identified parameters, as well as extensive simulations, reveal that QCM has in some cases desirable noise filtering properties.

1. Introduction

Accurate models are essential in analyzing systems under various excitations, boundary conditions and parameter changes. Analytical models, obtained by finite element or any other method, inevitably deviate from the true model due to uncertainties in geometry, material properties, boundary conditions, etc. Consequently, natural frequencies and modeshapes that are extracted from the test results do not agree with the predicted values from the analytical model. Model updating is the process of using the test results to correct the model so that it agrees, either completely or approximately, with the experimental data. In broader terms, it can be described as fusion of two sources of information, both inaccurate (the test results themselves contain errors resulting from measurement noise) and possibly incomplete, to obtain a better model.

A wide variety of model updating methods have been suggested, and a comprehensive survey of them can be found in [1]. Theoretically one can apply formal system identification and completely disregard the analytical model. Most approaches, however, do use the analytical model, as the eigendata obtained experimentally is not complete and cannot be the sole source of a model. Some methods, e.g. [2], are based on the complete frequency response function (FRF). A branch of works, e.g. [3,4], point to the similarity between model updating and eigenvalue assignment. These methods operate on a general matrix second-order model as seen in [3] where and eigenvalue-embedding technique is used in the presence of damping.

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The starting point for most model updating methods is the eigenvalue equation that should be satisfied. However, the notion of satisfying the equation is not unique and has three main interpretations. Sensitivity methods, e.g. [5,6], aim at matching the *solution* of the equation, i.e. natural frequencies and modeshapes, with the experimental data. The dependence of the natural frequencies and mode shapes on the physical parameters is calculated either numerically or analytically.

In Reference Basis methods [7–12], *perfect satisfaction* of the eigenvalue equation, with the measured data, is required. With certain parameters that are assumed to be accurate, the minimal deviation of the free quantities from their analytical values is sought. The main advantage of the method is its mathematical and numerical convenience. The main disadvantages in its original formulations [7,8] are that they do not account for any dependence on physical parameters, and therefore do not preserve the connectivity of the stiffness matrix. A variation of the method that retains the zero entries of the stiffness matrix was presented in [9]. This was done by using element-by-element multiplication laws with a structured relative perturbation matrix. In [10], the problem is looked at from a projection point of view. The projector is determined by the test data, and least squares solution is sought for the true unknown stiffness matrix, which preserved the correct connectivity. A different approach was taken in [11] that introduced the Generalized Reference Basis. Freedom in selecting the weighting matrix was added, enabling indirect influence over the outcome of the optimization. This added flexibility leads to the *a posteriori* connectivity assignment procedure suggested in [12].

A third approach is Minimization of the Error in the Constitutive Equations (MECE). The approach is used also in other related problems such as expansion and error localization [13,14]. In physical terms, MECE methods minimize the difference between the static stiffness forces and the dynamic inertia forces [14]. The main difference between the various methods in this class, e.g. [13–15], is the weighting of the error (residual force) that is used for the minimization.

The issue of measurement noise has been addressed explicitly in statistical terms in several works, e.g. [16–18]. Even in cases where there is no direct reference to noise, it appears indirectly through the use of least squares whose main role is noise reduction (least squares will not correct wrong or incomplete parameterization, nor any systematic error). However, the different weighting matrices in MECE are usually selected based on other considerations.

Quadratic Compression Method (QCM) was first introduced in [19] and was further developed in [20]. It is based on minimization of the error in a squared down, symmetric, version of the eigenvalue equation, and as a result deals with matrices of lower dimensions. While the actual QCM algorithm is different, the derivation in [20] showed that it can be interpreted as an MECE with a particular structure, with an effective weighting matrix that is closely related to the measurement noise. It was also shown there that the method exhibits superior filtering properties.

The presentation of QCM in [19,20] was restricted to stiffness updating, while the mass matrix was assumed to be known exactly. Even though this is a realistic scenario in many cases, because the stiffness is more susceptible to modeling errors, in many other cases the mass matrix needs to be updated as well. This paper presents the Quadratic Compression Method with both mass and stiffness updating. It is shown that the problem can be solved in two separate steps, independent of each other. The effect of noise is also investigated, and under some conditions, the favorable filtering properties of QCM are maintained.

2. Preliminaries

2.1. Problem statement

The general problem that model updating attempts to solve can be stated as follows: let the 'true' system be defined as

$$M_T\ddot{x}(t) + K_T x(t) = f(t) \quad x \in R^n, \tag{1}$$

where M_T is the true mass matrix, K_T is the true stiffness matrix, $x(t)$ is the displacement vector, $f(t)$ is any external force to the system, and n represents the number of degrees of freedom. The absolute accurate system is non-linear, has infinite degrees of freedom, etc. Thus 'true' refers to a system that is accurate enough for all practical purposes. The analytical model is given by

$$M_A\ddot{x}(t) + K_A x(t) = f(t) \quad x \in R^n, \tag{2}$$

where M_A and K_A are the analytical mass and stiffness matrices, respectively, obtained typically from a finite element model. The result of a modal test is a subset of $m<n$ experimental natural frequencies ω_{Ei} and experimental mode shapes ϕ_{Ei} that are arranged in the matrices

$$\Omega_E = diag(\omega_{Ei}^2), \quad \Phi_E = [\phi_{E1} \ldots\ldots \phi_{Em}].$$

Model updating procedures try to combine the analytical information (M_A, K_A) and the experimental results (Ω_E, Φ_E) to obtain a model that is closer to (M_T, K_T). Let σ_k, $k = 1, \ldots, p$ and α_j, $j = 1, \ldots, q$ be scalar parameters of the stiffness and mass matrices, respectively, arranged in the vectors $\sigma \in R^p$ and $\alpha \in R^q$, and assume for simplicity that the nominal value is $\sigma = \alpha = 0$. For comparison with methods that identify all the parameters simultaneously, we define the stacked vector of all