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Transcendental inverse eigenvalue problems in damage parameter estimation

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ABSTRACT

In this paper the author has introduced transcendental eigenvalue problems for estimating the damage parameters in the continuous structure from measured eigenvalues or natural frequencies. For simplicity axially vibrating rods are considered in which single or multiple damage parameters due to open cracks or notches are simulated by linear springs that are representative of the loss in stiffness or axial rigidity at the location of cracks. Transcendental eigenvalue problems (direct and inverse) associated with rod having multiple damage parameters have been formulated and solved. The numerical method for solving such eigenvalue problems are developed which overcomes the requirements of closed form characteristic frequency equations that are often unavailable. The modeling and solution approach developed here is utilized for evaluating the spectrum of rods with multiple damage parameters as well as for identifying the locations and severity of the damage parameters purely from the eigenvalues. Numerical examples and simulations corresponding to various damage configurations are presented and verified against experimental evidence. It is demonstrated that the solution of transcendental inverse eigenvalue problems can be successfully used for estimating the damage parameters by using only few and selected eigenvalues corresponding to the measured resonant and anti-resonant frequencies of the rod.

1. Introduction

Vibration based damage identification techniques are frequently used for the structural health monitoring of mechanical and civil structures. The problem of estimating the damage parameters (size of the cracks and its location) based upon spectral data i.e. eigenvalues (natural frequencies) or eigenvectors (mode shapes) have been well studied in the past and are summarized in the review papers by Dimarogonas [1], Salawu [2], Doebling et al. [3], Yan et al. [4], and Friswell [5]. Such problems can be divided into two categories: "*direct or forward problems*" and "*inverse problems*". The forward problems deal with estimating the spectral behavior of the damaged structure by knowing its mathematical model and comparing the data set with the baseline data of the undamaged structure. The change in spectral behavior, i.e. shift in natural frequencies or change in the mode shapes, corresponding to the different severity levels and damage locations, are used for estimating/identifying the damage. On the other hand, an associated inverse problem deals in estimating the

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location and severity of the damage based upon spectral data set that can be obtained by dynamic testing such as modal analysis. Relationships between the eigenvalues and eigenvectors of the damaged continuous system with the damage parameters have been studied by various researchers in the past [6–12].

Frequently, the damage parameter, open notch or crack, in a rod is simulated by linear springs with stiffness representing the loss in axial rigidity due to the damage. The stiffness parameter depends upon the size and shape of the crack and can also be related to the stress intensity due to the cracks/notches [13]. The formulation based on such approximation has been widely used in the past and some closed form solutions, relating the characteristic frequency equations and the location and severity of single damage parameter, have been obtained for the case of axially vibrating rods and transversely vibrating beams [8]. It has been shown that the ratio of the lower two natural frequencies of the undamaged and damaged rod can identify the location of the crack. These results were further extended by Morassi and his colleagues [14–16] and they have shown that for some boundary configurations the ratios between the variations of the 2mth and mth resonant frequencies can determine the positions as well as stiffness of the spring representing the severity of the damage. In a recent paper by Dilena and Morassi [17], the authors have shown that the combination of resonant and anti-resonant frequencies can avoid the non-uniqueness of the damage location problem due to the symmetry of the structure. In the recent edition of his manuscript, Gladwell [18] has compiled the research associated with the damage identification in the continuous systems and also formulated associated inverse eigenvalue problems. It is shown that although inverse eigenvalue problems related to damage identification may lead to ill-posed problems, however by carefully selecting spectral data set, such inverse problems can be formulated as well-posed problems and can be solved leading to unique solutions.

In few recent studies [14,19–26], spectral behavior of structures with two or more damage parameters (multiple springs) were also studied and it was observed that the characteristic frequency equations of the rod with multiple damage parameters are complicated and lend themselves to large determinant equations similar to those presented in [20,21]. The characteristic frequency equations corresponding to single or multiple damage states are often used for solving associated forward problems by using various numerical methods such as modified Fourier series methods [22,23], transfer matrix approach [24], recurrence scheme in [25] and smooth function method [26]. However obtaining the closed form characteristic frequency equations for an arbitrarily large number of damage parameters is not feasible.

It is convenient to analyze a continuous system with multiple damage parameters by using associated discrete models obtained from finite element or finite difference approximation techniques. The finite dimensional discrete models lead themselves to the solutions of algebraic eigenvalue problems for evaluating the spectrum of damaged structures [27,28]. However such discrete models introduce modeling errors, and, hence the spectrum obtained after solving algebraic eigenvalue problems, irrespective of the model order used, do not actually represent the spectrum of the continuous system [29,30]. In recent years finite element based modal updating techniques are being used in structural health monitoring for identifying the single and multiple damage patterns by using finite dimensional discrete models [31–34]. It is arguably the most successful practice in engineering for damage identification. However these techniques are also beset by the requirements of large vibration data set, sensitivity of measured eigenvectors, discretization errors, the requirements of a fine mesh and spillover phenomenon [35,36]. It is important to note here that in this paper the objective is not to highlight the deficiencies of the existing damage detection techniques that are based upon discrete finite element models, rather to present an alternative approach in which such discretization can be avoided, a better qualitative analysis of a damage structure can be obtained, and identification of multiple damage parameters can be realized based upon only few measurable natural frequencies. Identification based on natural frequencies is preferred because they can be measured with ease and are more robust in comparison to measured eigenvectors or mode shape. The key issue of accurately and simultaneously identifying the severity and location of the multiple damage parameters purely from the few measured eigenvalues is addressed here.

In this paper, an alternate modeling and solution strategy for forward and inverse eigenvalue problems associated with continuous systems with multiple damage parameters (open cracks and notches) is introduced here in a tutorial setting, which avoids discrete modeling and utilizes only few eigenvalues for damage parameter estimation. For simplicity, axially vibrating rods with multiple damage parameters simulated by linear springs are formulated here leading to transcendental eigenvalue problems (TEP). The spectrum of damaged rods is obtained by using a recently developed algorithm for the solution of TEP [37–39], hence circumventing the evaluation of large and complicated characteristic frequency equations. A new transcendental inverse eigenvalue problem (TIEP) is also introduced. With numerical examples and verification studies, it is demonstrated that by solving the TIEP, the location and severity of damage parameters can be identified accurately from the few and selected eigenvalues of the rod. In the next section, the formulation for axially vibrating rods, with multiple intermediate springs simulating multiple damage parameters, is presented, leading to TEP. In Section 3 numerical strategies for solving TEP and TIEP are presented. Several examples are presented in Section 4 associated with the forward and inverse eigenvalue problems. The spectrum of the damaged rod is obtained by solving TEP and they are compared with available experimental data from other authors. The effect of the location and severity of the damage on the spectrum is studied. By solving TIEP the severity and location of multiple damage parameters are identified simultaneously by using both simulated and/or measured eigenvalues. The research is summarized and the scope of future research is concluded in Section 5.