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Prediction and assignment of latent roots of damped asymmetric systems by structural modifications

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A B S T R A C T

This paper studies the latent roots of damped asymmetric systems in which the stiffness matrix is asymmetrical. The asymmetric terms are due to 'external' loads and are represented by a parameter or parameters. The latent roots of such asymmetric systems are complex and the real parts become positive at some critical values of the parameter(s) (critical points). The work reported in this paper consists of two parts. The first part presents a method for predicting the latent roots of the damped asymmetric system from the receptance of the damped symmetric system. The second part presents an inverse method for assigning latent roots by means of mass, stiffness and damping modifications to the damped asymmetric system again based on the receptance of the *unmodified* damped *symmetric* system. The simulated numerical examples of a friction-induced vibration problem show the complexity in assigning stable latent roots for damped asymmetric systems. It is found that it is quite difficult to assign the real parts of latent roots to stabilise the originally unstable asymmetric system and sometimes there is no solution to the modification that is intended to assign certain latent roots.

1. Introduction

The mass matrix and stiffness matrix of engineering structures can be assumed to be symmetrical, respectively, positive definite and semi-positive definite in general. The damping matrix of these structures is also symmetrical and at least semi-positive definite. The eigensolutions of such structures are well studied and the vibration of such structures is stable, apart from a few rigid-body modes when the stiffness matrix is only semi-positive definite. There are, however, engineering problems whose stiffness matrices are asymmetrical. Usually asymmetry is produced not by the structure itself, but by some external loads [1] interacting with the structure, such as friction in brakes noise problems [2] or airflow in aeroelastic flutter problems [3]. These external loads acting on a structure are considered to be internal to the system that contains the structure. In this paper, a structure is understood to be a building or a machine while a system is considered to include a structure and the forces acting on the structure.

In linear or linearised models of friction-induced vibration, friction introduces asymmetric terms into the stiffness matrix [4–11]. Hoffmann et al. [6] demonstrated that as an off-diagonal stiffness element proportional to the friction coefficient increased two pairs of purely imaginary eigenvalues would coalesce (the two frequencies became identical) at a critical value and become complex with one pair having positive real parts above this critical value in their two-degrees-of-freedom undamped system with sliding friction. Hoffman and Gaul [10] went on to show that viscous damping could

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destabilise the friction-induced vibration of a similar two-degrees-of-freedom system. The perturbation method presented by Huang et al. [8] was able to predict complex eigenvalues and the critical friction coefficient from the frequencies and modes of the symmetric system. However the formulas are very complicated. Flutter instability in the form of complex eigenvalues with positive real parts is assumed to signify squeal noise in frictional brakes. The standard approach for brake squeal analysis in automotive industry [2] is computing many complex eigenvalues at gradually varying values of interesting system parameters [12]. Needless to say this is a very expensive process. This paper presents a method for determining the latent roots of the damped asymmetric system based on the receptance of the damped symmetric system, which is easier to obtain.

The same method is developed to assign latent roots of the damped asymmetric system by means of mass, stiffness and damping modifications to the asymmetric system when it is unstable. As the eigenvalue problem and the receptance of the symmetric system are usually well understood and much easier to solve and even to measure than the asymmetric system, this receptance-based method has a big advantage. The idea of determining the solutions of a modified system from the receptance of the original system has been used in the context of structural modifications [13–16] and structural vibration control [1,17]. However, assignment of the real parts of latent roots of asymmetric systems proves more difficult to achieve. Shi et al. [18] performed structural design optimisation for suppressing disc brake squeal (believed to be the result of friction-induced vibration), in which 46 1 g masses were evenly distributed on the back of the friction pads and they were able to shift all the unstable latent roots (complex eigenvalues) into the left half of the complex eigenvalue plane.

Chu [19] recently presented a robust algorithm for pole assignment of second-order damped systems with a symmetric stiffness matrix through feedback control. Chu also outlined the approach to pole assignment for damped asymmetric systems when there are no degenerate eigenvalues. While the poles in that study were all stable (with negative real parts), this paper deals with unstable latent roots (equivalent to poles in control theory) and re-assigns them by structural modifications to produce a stable system.

2. Asymmetric stiffness matrices

The free vibration problem of a structure may be written as

$$(\mathbf{K} + \lambda^2 \mathbf{M})\boldsymbol{\varphi} = 0 \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are mass and stiffness matrices. When damping is considered, the free vibration problem becomes

$$(\mathbf{K} + \lambda \mathbf{C} + \lambda^2 \mathbf{M})\boldsymbol{\varphi} = 0 \tag{2}$$

where $\mathbf{C}$ is damping matrix. Usually, λ^2 in Eq. (1), which is a linear eigenvalue problem, and λ in Eq. (2), which is a quadratic eigenvalue problem, are usually both called an eigenvalue. To add confusion, if $\mathbf{K}$ is asymmetric, λ can be complex. In order to help subsequent discussion, λ and $\boldsymbol{\varphi}$ in both equations are called the latent root and latent vector, respectively. This terminology was due to Lancaster [1].

Two typical ways of introducing friction into a system were shown in [2,5,6]. It is clear that both ways introduce asymmetry in the stiffness matrix. As friction coefficient μ increases, the asymmetric terms become greater and there can be two scenarios whereby the latent roots gain positive real parts. In the first scenario, a pair of conjugate latent roots cross the vertical imaginary axis, as shown in the simulated examples later in the paper and also illustrated by Huseyin [20] two decades ago. In the second scenario, two conjugate pairs of the latent roots are getting closer and closer and eventually coalesce when the real parts become non-zero (half of them become positive and the other half become negative) [6]. Flutter instability sets in at this point, known as a critical point [20]. This mechanism whereby friction causes flutter instability has been known in the study of brake squeal as mode-coupling [8–10]. Huseyin [20] made extensive studies of stability of various types of low-degrees-of-freedom dynamic systems. The focus in this paper is on damped systems with an asymmetric stiffness matrix.

As the major issue in asymmetric systems is the lack of stability, assignment of the real parts to produce stable systems is much more important than assignment of frequencies, which has been studied extensively for symmetric systems. The asymmetry comes from the interaction between the external loads and the structure involved and the problem is self-excited vibration. As external excitation is usually not a big issue in self-excited vibration problems, assignment of frequencies in these problems are not as important as assignment of the real part to steer an unstable system into the stable territory. This proves to be quite difficult as simulated examples presented later in the paper will show. The main reasons are due to the fact that the real part of a latent is insensitive to the change in mass and stiffness while the imaginary part is insensitive to change in structural damping. As a first attempt to assign latent roots for the sake of system stability, only simple, local mass, stiffness or damping modifications are explored in the paper. More sophisticated techniques of structural modifications can be found in [13,16,21,22].

3. Prediction of latent roots

It is fairly easy to determine the latent roots of a simple system with a small number of degrees-of-freedom, like the one studied in [6,10], which admits close-form solutions. For systems with a fairly large number of degrees-of-freedom,