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Parametric identification of a time-varying structure based on vector vibration response measurements ☆

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ABSTRACT

The problem of parametric output-only identification of a time-varying structure based on vector random vibration signal measurements is considered. A functional series vector time-dependent autoregressive moving average (FS-VTARMA) method is introduced and employed for the identification of a "bridge-like" laboratory structure consisting of a beam and a moving mass. The identification is based on three simultaneously measured vibration response signals obtained during a single experiment. The method is judged against baseline modelling based on multiple "frozen-configuration" stationary experiments, and is shown to be effective and capable of accurately tracking the dynamics. Additional comparisons with a recursive pseudo-linear regression VTARMA (PLR-VTARMA) method and a short time canonical variate analysis (ST-CVA) subspace method are made and demonstrate the method's superior achievable accuracy and model parsimony.

1. Introduction[1]

Many engineering structures, such as traffic-excited bridges [1,2], cranes [3], robotic devices and flexible mechanisms [4,5], rotating machinery [6,7], variable geometry aerospace structures [8] and so on, exhibit characteristics that vary with

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Abbreviations: AR, autoregressive; ARMA, autoregressive moving average; BIC, Bayesian information criterion; CVA, canonical variate analysis (method); FS-VTAR, functional series VTAR (model); FS-VTARMA, functional series VTARMA (model); GA, genetic algorithm; NID, normally independently distributed; MA, moving average; OLS, ordinary least squares (method); PE, prediction error (method); PLR, pseudo-linear regression (method); PLR-VTARMA, PLR-estimated VTARMA (model); RSS, residual sum of squares; SPWV, smoothed pseudo-Wigner–Ville (method); ST-CVA, short time CVA (method); VTAR, vector time-dependent AR (model); VTARMA, vector time-dependent ARMA (model); 2SLS, two stage least squares (method).

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[1] Important conventions and symbols:

- Bold-face upper/lower case symbols designate matrix/column–vector quantities, respectively.
- Matrix transposition is indicated by the superscript T.
- A functional argument in parentheses designates function of a real variable; for instance $x(t)$ is a function of analog time $t \in \mathfrak{R}$.
- A functional argument in brackets designates function of an integer variable; for instance $x[t]$ is a function of normalized discrete time $(t = 1, 2, \ldots)$. The conversion from discrete normalized time to analog time is based on $(t-1)T_s$, with T_s standing for the sampling period. A time instant used as superscript to a function indicates the set of values of the function up to that time instant; for instance $x^t \triangleq \{x[i], i = 1, 2, \ldots, t\}$.

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time. These structures are referred to as *time-varying* (TV) or *non-stationary*. Structures with variability due to changing geometric configuration form an important subclass of TV structures. In case that the geometric configuration changes continuously with time, the term *continuously variable configuration* structure is used. Robotic devices, cranes, and deployable structures constitute primary examples of this subclass.

The problem of TV structural identification involves the determination of a mathematical model representing the underlying TV dynamics based on excitation and/or vibration response signals. In contrast to time-invariant (TI) structures, which produce *stationary* vibration responses with time-invariant statistical characteristics, the responses of TV structures are *non-stationary*, characterized by time-dependent statistical characteristics [9, 10, pp. 52–55]. TV structural identification thus is a rather complicated issue, in which non-stationary signal analysis techniques need to be employed [9–12].

In many occasions, such as in railway bridges, aircraft, surface vehicles and so on, controlled testing may not be feasible under realistic operating conditions, so that structural identification is pursued by exclusively using measured vibration response signals. This is the *output-only TV identification problem* which is in the focus of the present study.

Output-only TV structural identification methods are classified as *non-parametric* or *parametric*. The former are based on non-parametric representations of the non-stationary response as a simultaneous function of time and frequency (*time–frequency representations*). The short time Fourier transform (STFT) [13], the Cohen class of distributions [14], and wavelet based methods [15] constitute some of the main and most frequently used methods. Non-parametric methods are easy to use, but lack in terms of representation parsimony (economy), frequency resolution, tracking accuracy—especially in cases of fast variations in the dynamics—and flexibility in analysis [4,9,11].

Parametric methods are based on time-dependent autoregressive moving average (TARMA) and time-dependent state space models which may be thought of as conceptual extensions of their conventional, stationary, counterparts, in that their parameters are time-dependent (for instance see [9,10]). Parametric models and the corresponding identification methods may be further classified according to the type of mathematical structure imposed on the evolution of the TV model parameters as *unstructured*, *stochastic*, or *deterministic* parameter evolution. The reader is referred to the recent survey by Poulimenos and Fassois [9] for a detailed account and assessment of various parametric methods.

The majority of parametric TV structural identification methods are limited to the simple *single* vibration response signal (univariate) case. Although the *multiple* vibration response (vector or multivariate) case is much more important from a practical standpoint, it has thus far received limited attention. Yet, vector identification can lead to much more complete descriptions, reduced data acquisition and processing times, improved data set "consistency", and also improved modal parameter accuracy [16].

The majority of available vector parametric output-only methods are of the *unstructured parameter evolution* type, which imposes no mathematical structure upon the evolution of their TV parameters. Kirkegaard et al. [17] use recursive vector TARMA models for the identification of the TV parameters of a simulated reinforced concrete structure under earthquake excitation. A similar identification problem is considered by Yang et al. [18,19], who use recursive least squares estimation based on the physical model of the structure and adaptive forgetting factors. Various recursive and short time subspace methods have been also used for vector TV structural identification over the last decade. Although most of them have been limited to the analysis of simulated TV structures (for instance [20,21]), Goethals et al. [22] and Mevel et al. [23] use a recursive and a short time subspace method, respectively, in order to track the TV modal characteristics of an aircraft during flutter vibration. Bosse et al. [24] consider the identification of a TV truss structure via a recursive subspace based algorithm. A subspace method is also utilized by Liu and Deng [12] in order to capture the TV dynamics of an axially moving cantilever beam. In contrast to the previously mentioned studies, their analysis requires a significant number of signal realizations (and thus experiments). It should be additionally remarked that unstructured parameter evolution methods may not be suitable for fast varying structures, while also achieving low representation parsimony (economy).

Noticeably fewer studies are available on vector TV structural identification using *stochastic parameter evolution methods*, which impose stochastic mathematical structure upon the evolution of the TV model parameters through stochastic smoothness constraints. Kitagawa et al. [25, pp. 172–174; 26] use such methods for the modelling and analysis of earthquake ground motion signals, while the modelling of wind speed time histories is considered by Chen [27]. Stochastic parameter evolution methods have also been used for the identification of structural variations of a bridge under construction and varying operating conditions [28], as well as for the identification, analysis and health monitoring of gearboxes [29,30]. Stochastic parameter evolution methods may offer improvements in accuracy and tracking over the unstructured parameter evolution methods, but model parsimony remains an issue.

The class of *deterministic parameter evolution methods* is known to offer a number of advantages and improved accuracy and model parsimony in the single signal (univariate) case [9], yet it has not been thus far used for vector TV structural

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(*footnote continued*)

- A hat designates estimator/estimate of the indicated quantity; for instance $\hat{\boldsymbol{\theta}}$ is an estimator/estimate of $\boldsymbol{\theta}$.
- $\mathscr{B}$ stands for the backshift operator defined such that $\mathscr{B}^i \cdot \boldsymbol{x}[t] \triangleq \boldsymbol{x}[t-i]$.