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Structural parameters identification using improved normal frequency response function method

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Abstract

An improved method that is based on a normal frequency response function (FRF) is proposed in this study in order to identify structural parameters such as mass, stiffness and damping matrices directly from the FRFs of a linear mechanical system. This paper demonstrates that the characteristic matrices may be extracted more accurately by using a weighted equation and by eliminating the matrix inverse operation. The method is verified for a four degrees-of-freedom lumped parameter system and an eight degrees-of-freedom finite element beam. Experimental verification is also performed for a free–free steel beam whose size and physical properties are the same as those of the finite element beam. The results show that the structural parameters, especially the damping matrix, can be estimated more accurately by the proposed method.

Keywords: Structural parameters; Normal FRF; Weighted equation; Matrix inverse elimination; Least squares method

1. Introduction

In order to predict accurate dynamic characteristics of a mechanical system, it is necessary to formulate a mathematical model as accurately as possible in terms of structural parameters such as mass, stiffness and damping matrices of the system. However, since most mechanical systems are complicated, it is difficult to accurately estimate their system parameters. For these reasons, many studies have been undertaken using theoretical and experimental methods in order to identify structural parameters.

One of the general methods for estimating system parameters is to indirectly use frequency response function (FRF) data. The system parameters are extracted using modal parameters such as natural frequencies and mode shapes derived from the FRF data [1,2]. However, the structural parameters estimated by these methods can be affected by several factors including the inaccuracy of FRFs, the errors in the extracted modal parameters and the incompleteness of the modal information [3].

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A more extensively studied method that is theoretically easier and simpler than the indirect method is known as a direct method. Using the direct method, the system matrices can be obtained directly from the measured FRFs of a mechanical system without using modal parameters. Fritzen [4] proposed the Instrument Variable (IV) method that is suitable for the estimation of structural parameters from noisy data. By comparing the estimated results from the IV method with those from the least squares method, Fritzen's work demonstrates that the IV method is less sensitive to measurement noise than the least squares method. However, the IV method requires numerous data points in order to estimate the structural parameters and it is therefore very time consuming. In order to overcome these problems, Wang [5] developed the weighted FRF that can be combined with the IV method. In Wang's approach, appropriate configurations of data distribution in FRFs were proposed in order to reduce the number of data points required by the IV method.

Chen et al. [6] developed the 'normal FRF method', which extracts the normal FRFs from the complex FRFs of a structure. Chen et al. demonstrated that accurate identification results can be obtained when the damping matrix is identified independently from the mass and stiffness matrices. Lee et al. [7] conducted a theoretical validation and related error analysis for the normal FRF method as developed by Chen et al. and developed a method for experimental estimation of damping matrices of a general dynamic system. Lee et al. applied that method to a single-reed beam with a viscous damper attached, and identified two types of damping mechanisms directly from the measured FRFs, i.e. viscous and hysteretic damping matrices. Later, they also developed a much simpler algorithm in numerical operations that identifies the damping matrices directly from the dynamic stiffness matrix (or the inverse of the frequency response matrix) [8]. However, good care must be taken of some technical issues, including a phase-matching technique between force transducer and accelerometers, a sign convention of FRFs, and a conditioning of frequency response matrix that makes the frequency response matrix symmetric.

In this paper, an improved method, which itself is based on the normal FRF method developed by Chen et al., is presented so as to identify the structural parameters directly from the FRFs of a mechanical system. Accurate results have been achieved by two modified and additional theoretical procedures, that of the elimination of matrix inverse operation and the imposition of weighting matrix. The advantage of this improved method is validated by two numerical examples, that of a simple four degrees-of-freedom (4 d.o.f.) lumped parameter system and that of an 8 d.o.f. finite element beam structure. Its robustness to noisy data are also illustrated through experimental verification of a steel beam. All the results from the present method have been compared with exact results and those of the normal FRF method.

2. Identification theory

The equation of motion for a vibratory system can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + (\mathbf{K} + \mathrm{j}\mathbf{D})\mathbf{x}(t) = \mathbf{f}(t), \tag{1}$$

where **M**, **C**, **K** and **D** represent the mass, viscous damping, stiffness and hysteretic damping matrices of the system, respectively, and $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$ and $\mathbf{f}$ are the vectors of displacements, velocities, accelerometers and forces, and $\mathrm{j} = \sqrt{-1}$. For harmonic excitation, Eq. (1) is expressed as

$$(-\omega^2\mathbf{M} + \mathbf{K})\mathbf{X}(\omega) + \mathrm{j}(\omega\mathbf{C} + \mathbf{D})\mathbf{X}(\omega) = \mathbf{F}(\omega). \tag{2}$$

Multiplying both sides of Eq. (2) by $[-\omega^2\mathbf{M} + \mathbf{K}]^{-1}$ yields

$$(\mathbf{I} + \mathrm{j}[-\omega^2\mathbf{M} + \mathbf{K}]^{-1}[\omega\mathbf{C} + \mathbf{D}])\mathbf{X}(\omega) = [-\omega^2\mathbf{M} + \mathbf{K}]^{-1}\mathbf{F}(\omega). \tag{3}$$

Here, **I** denotes the identity matrix. By introducing the normal FRF matrix $\mathbf{H}^N(\omega)$ and the transformation matrix $\mathbf{G}(\omega)$ as defined in Ref. [6] as

$$\mathbf{H}^N(\omega) = [-\omega^2\mathbf{M} + \mathbf{K}]^{-1}, \tag{4}$$

$$\mathbf{G}(\omega) = \mathbf{H}^N(\omega)[\omega\mathbf{C} + \mathbf{D}], \tag{5}$$

and substituting Eqs. (4) and (5) into Eq. (3) yields

$$[\mathbf{I} + \mathrm{j}\mathbf{G}(\omega)]\mathbf{X}(\omega) = \mathbf{H}^N(\omega)\mathbf{F}(\omega). \tag{6}$$