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Estimation of modal parameters using bilinear joint time–frequency distributions

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Abstract

In this paper, a new method is proposed for modal parameter estimation using time–frequency representations. Smoothed Pseudo Wigner–Ville distribution which is a member of the Cohen's class distributions is used to decouple vibration modes completely in order to study each mode separately. This distribution reduces cross-terms which are troublesome in Wigner–Ville distribution and retains the resolution as well. The method was applied to highly damped systems, and results were superior to those obtained via other conventional methods.

Keywords: Modal parameter estimation; Joint time–frequency distributions; Highly damped systems

1. Introduction

Numerous algorithms have been developed for estimating modal parameters using time and frequency domains separately over the past 30 years. In recent years, joint time–frequency (JTF) and wavelet transform (WT) methods have attracted many researchers to use great capabilities of these transforms.

Several papers have been published on applying WT for estimating modal parameters in recent years. Lardies and Gouttebroze [1] used WT and presented a special form of the Morlet Wavelet which gave rewarding results. Le and Argoul [2] studied three different wavelets and tackled the "Edge Effect" problem. Haase and Widjajakusuma [3] presented a method based on the WT for fault detection using modal parameters. On the other hand, there are researches in Biomedical Magnetic Resonance Spectroscopy which are very similar to the modal analysis. Serrai et al. [4] used the Morlet Wavelet and established an algorithm with less approximation in comparison with some other works.

However, to the best of the authors' knowledge, no work has yet been reported on estimating modal parameters using bilinear JTF distributions. These distributions expand the energy of a signal concurrently in

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time and frequency. This makes them a natural choice for fully decoupling the modes of vibration which happen at distinct frequencies.

Wigner–ville distribution (WVD) which is a member of Cohen's class distributions has the best resolution in time–frequency plane among all time–frequency representations of this class. But it has a deficiency of so-called cross-terms. Cross-terms appear when a signal has two or more components in the time–frequency plane. In a free decay response signal, these components are different modes of vibration. Thus, WVD is incapable of giving an appropriate discrimination of these components in the time–frequency plane in general. In order to compensate this drawback, dozens of joint time–frequency representations have been introduced during last decades [5–8].

Smoothed Pseudo Wigner–Ville (SPWV) is a member of Cohen's class distributions which utilizes two different smoothing windows on WVD, separately in time and in frequency, and eliminates cross-terms considerably [6].

In this paper, we introduce a method for the estimation of modal parameters using the SPWV distribution. Free decay response of a linear mechanical system with proportional damping ratios is used to estimate modal parameters (natural frequency and damping) of the system. It can be applied to single-degree-of-freedom (sdof) and multi-degree-of-freedom (mdof) linear systems. The exact analytical time-frequency distribution of a one-dof system is obtained and modal parameters are extracted from the formulation. Afterward, in view of the fact that SPWV decouples vibration modes in the time-frequency domain, results are expanded to a mdof system. In order to demonstrate the capability of the proposed method, the damping ratio estimation of a 2dof highly damped system is examined and compared to two other methods.

The paper is organized as follows. Section 2 is a brief introduction to Cohen's class distributions and SPWV characteristics. In Section 3, WVD of the free decay response of a sdof system is obtained. The SPWV of free decay response of a sdof system is derived in Section 4. Section 5 generalizes the results of the sdof system to mdof systems considering the fully decoupling of vibration modes in time–frequency domain. In Section 6, two examples are given and fully discussed. Finally concluding remarks are given in Section 7.

2. Cohen's class

The representations that describe a signal's frequency behavior fall predominantly into two categories [5]: linear representations such as the Fourier transform, and quadratic representations such as the power spectrum (PS). Quadratic representations can be viewed as distributing signal's energy into frequency, time–frequency, or time-scale variables. In this section we introduce a counterpart to the power spectrum: the quadratic joint time frequency representation known as Cohen's class distributions. The main core of all TFR is the Wigner–Ville Distribution (WVD) which is defined as:

$$\mathrm{WVD}_x(t,f) = \int_{-\infty}^{\infty} x\left(t+\tau/2\right)x^*\left(t-\tau/2\right)\mathrm{e}^{-j2\pi f\tau}\,\mathrm{d}\tau. \tag{1}$$

WVD can be seen as the instantaneous version of the power spectrum [6]. Theoretically, the WVD has the best time–frequency resolution among all time-frequency representations. But it does suffer from the serious problem of cross-terms, which occurs when the signal has two or more distinct time–frequency features [7].

In order to overcome this deficiency, other bilinear JTF distributions have been developed over the last decades such as the Pseudo WVD (PWVD), Smoothed Pseudo WVD (SPWVD) [6], Choi-Williams distribution (CWD) and cone-shape distribution (ZAMD) [5]. It is interesting to note that all these bilinear representations can be written in a general form that was introduced by Cohen [5]. The discovery of the general form of bilinear TFR facilitates us with the design of the desired TFR.

This general form can be written as

$$\begin{aligned} T_x(t,f) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi(t-s,f-\eta)x\left(s+\frac{\tau}{2}\right)x^*\left(s-\frac{\tau}{2}\right)\mathrm{e}^{-j2\pi\eta\tau}\,\mathrm{d}\tau\,\mathrm{d}s\,\mathrm{d}\eta \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi(t-s,f-\eta)WVD_x(s,\eta)\,\mathrm{d}s\,\mathrm{d}\eta, \end{aligned} \tag{2}$$