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Natural frequency and mode shape analysis of structures with uncertainty

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Abstract

Two methods called random factor method (RFM) and interval factor method (IFM) for the natural frequency and mode shape analysis of truss structures with uncertain parameters are presented in this paper. Using the RFM, the structural physical parameters and geometry can be considered as random variables. The structural stiffness and mass matrices can then, respectively, be described by the product of two parts corresponding to the random factors and the deterministic matrix. The structural natural frequencies, mode shapes and random response can be expressed as the function of the random factors. By means of the random variable's algebra synthesis method, the computational expressions for the mean value and standard deviation of natural frequencies and mode shapes are derived from the Rayleigh quotient. Using the IFM, the structural parameters can be considered as interval variables and the computational expressions for the lower and upper bounds of the natural frequency and mode shape are derived by means of the interval operations. The effect of uncertainty of individual structural parameters on structural dynamic characteristics, and the comparison of structural natural frequency and mode shape using the RFM and IFM are demonstrated by truss structures.

Keywords: Uncertainty; Random factor method; Interval factor method; Natural frequency; Mode shape; Truss structures

1. Introduction

The probabilistic and non-probabilistic interval analyses of structures with uncertain parameters are very significant research fields in engineering [1–3]. Typical engineering structures include bridges, buildings, offshore structures, vehicles, ships, and aerospace structures, etc. These structures can be described as stochastic or interval (bounded uncertain), due to variability in their geometric or material parameters, or uncertainty resulting from the assembly process and manufacturing tolerances. Stochastic or interval structures may correspond to the uncertainty of a single structure (for example, a bridge, offshore platform, antenna or space structure) or a batch of nominally identical structures (e.g. vehicles leaving the production line).

The most common approach to problems of uncertainty is to model the structural geometric and material parameters as random variables. Under these circumstances, the mean value, variance, standard deviation of

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individual structural parameter and the correlation between different structural parameters are provided by the probabilistic information (probability density function and joint probability distribution function) of the structural parameters. However, the probabilistic approaches cannot give reliable results unless sufficient experimental data or statistical information are available to validate the assumptions about the joint probability densities of the random variables or functions involved [4]. In some cases, we can only obtain the range or the lower bound and upper bound of the structural parameters. Therefore, the probabilistic analysis method is not available if the standard deviation or probability density function cannot be obtained, and the interval analysis method will be very useful. In the interval analysis method, a bounded uncertain structural parameters can be described as an interval variable and only its midpoint value (mean value), lower and upper bounds are required.

For the dynamic characteristics analysis of structures with random parameters, the Monte-Carlo simulation method (MCSM) [5,6] and perturbation method [7,8] are widely used. Since the mid-1960s, a new method called the interval analysis has appeared. Moore [9] and Alefeld [10] have done the pioneering work. Rohn [11] has studied the standard interval eigenvalue problem of a symmetric interval matrix and derived formulas for interval eigenvalues when the error matrix has rank one. Hudak [12] has investigated ways to relate this to the eigenvalues of a constant matrix under certain conditions. Hansen [13] in his book discussed the global optimisation using interval analysis. Mathematically, linear interval equations, non-linear interval equations and interval eigenvalue problems in the method have been resolved. However, because of the complexity of the algorithm, it is difficult to apply these results to practical engineering problems. Recently, McWilliam [14], Chen et al. [15] and Qiu et al. [16] have investigated the structural responses and eigenvalue problems of structures with uncertain parameters. In their studies, several important results have been obtained by using interval analysis and matrix perturbation techniques.

In the Monte-Carlo method, the values of a structural parameter are changed within a given range. A large amount of dynamic analyses on the same structure is then performed, and the statistical data (mean value and standard deviation) of the natural frequencies and mode shapes are obtained. However, the MCSM needs a large amount of computational work to obtain the dynamic characteristics of random structures, and it becomes difficult to analyse large-scale structures due to the intensive computational effort required. Thus, the MCSM is frequently chosen as a scheme to verify the correctness of a new methodology. The perturbation method uses a combination of matrix perturbation theory, finite element method and Taylor series expansion to obtain the dynamic characteristics of structures with uncertainty. In perturbation methods, structural mass matrix $[M]$ and stiffness matrix $[K]$ are, respectively, expressed as their mean (deterministic) values $[\bar{M}]$ and $[\bar{K}]$ plus their perturbation values $\varepsilon_1[\bar{M}]$ (or $\Delta[\bar{M}]$) and $\varepsilon_2[\bar{K}]$ (or $\Delta[\bar{K}]$), that is, $[M]=[\bar{M}]+\varepsilon_1[\bar{M}]$ and $[K]=[\bar{K}]+\varepsilon_2[\bar{K}]$. The uncertainty of all the structural parameters is expressed as perturbation values in the structural mass and stiffness matrices. These perturbation values are not random variables or interval variables, but simply small values. Therefore, the perturbation method cannot reflect the effect of the change of the individual structural parameters on the natural frequency and model shape of a structure.

In this paper, the random and interval dynamic characteristics analysis of structures with uncertain parameters are investigated, and two methods called random factor method (RFM) and interval factor method (IFM) are presented. Truss structures are used to illustrate examples of these methods, in which structural physical parameters (Young's modulus and mass density) and geometry (length and cross-sectional area of bar) are considered as random variables or interval variables.

The main idea of RFM has been used to analyse the dynamic characteristics and response of structures with random parameters [17,18]; however, the randomness of the mode shapes was neglected because we thought the randomness of dynamic characteristics (natural frequency and mode shape) can be represented by the randomness of natural frequency in the past. In fact, natural frequency and mode shape are all random variables, and their randomness must be considered simultaneously if the structural parameters are random variables.

The procedure for the RFM is as follows. Firstly, a structural parameter variable with uncertainty is expressed as a random factor multiplied by the mean value of this structural parameter. Secondly, the structural mass and stiffness matrices are expressed as random factors of the structural parameters multiplied by their deterministic values (mean values), respectively. Finally, the dynamic characteristics are expressed as the functions of these random factors. Likewise, in IFM, a structural parameter interval variable is expressed as an interval factor multiplied by the mean value (midpoint value) of this structural parameter. Secondly, the