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Time-fractional effect on pressure waves propagating through a fluid filled circular long elastic tube

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ABSTRACT

The pressure waves propagating through an incompressible inviscid fluid that moves in a circular cylindrical long elastic tube are considered. The reductive perturbation method is used to derive the KdV equation from the hydrodynamic equations of the system. The Euler–Lagrange variational technique described by Agrawal has been applied to formulate the time-fractional KdV equation. The derived time-fractional KdV equation is solved by employing the variational-iteration method represented by He. The effects of the tube and fluid parameters and the time fractional order on the propagation of pressure waves are investigated.

1. Introduction

The propagation of pressure waves in fluid that moves in large vessels was studied by many authors, e.g. References 1–16. Many of the works studied the small amplitude wave propagation in elastic tubes, ignored the nonlinear effects and focused on the dispersive characteristic [3–5]. When the nonlinear character appears, either finite amplitude or small-but-finite amplitude wave is considered, depending on the nonlinearity order. The propagation of finite waves through fluid filled elastic or viscoelastic tubes was studied [6–9]. Also, the small-but-finite amplitude waves propagating in distensible tubes were investigated [10–16], where the Korteweg–de Vries (KdV) equation appears due to the balancing between the nonlinearity and the dispersion effects.

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The KdV equation as evolution and interaction model of nonlinear waves is employed to represent a wide range of physical phenomena. The KdV equation was first formulated as an evolution equation governing one-dimensional, small amplitude, long surface gravity waves propagating in a shallow water channel [17]. Afterward, the KdV equation has appeared in a number of other physical problems such as ion-acoustic waves, collision-free hydro-magnetic waves, plasma physics, stratified internal waves, lattice dynamics, etc. [18]. By means of the KdV model, some theoretical physics phenomena in quantum mechanics domain and continuum mechanics for shock wave formation are explained. The KdV model is also applied in many scientific fields like fluid dynamics, aerodynamics, solitons, turbulence, boundary layer behavior and mass transport.

Most of the forces in nature are non-conservative: dissipative and/or dispersive forces. The classical mechanics treated conservative forces using integer differential equations, while the non-conservative forces can be described in terms of the non-integer differential equations. Non-integer differentiation and integration is called Fractional Calculus, which is a field of mathematics study that generalizes the traditional definitions of calculus integral and derivative operators. During the last decades, Fractional Calculus has acquired importance due to its applications in various fields of science and engineering, including electrical networks, signal processing, optics, fluid flow, viscoelasticity, rheology, probability and statistics, dynamical process in self-similar and porous structures, diffusive transport, control theory of dynamical systems, electrochemistry of corrosion, and so on [19–35].

Riewe [19,20] used fractional derivatives [21–23] in the action to have non-conservative Euler–Lagrange equations. In terms of Riemann–Liouville fractional derivatives, Agrawal [24–26] used variational technique to formulate fractional equations of motion. These Euler–Lagrange equations are employed to investigate different real problems [27–35].

The fractional differential equations are solved by applying several methods such as: Fourier transformation method, Laplace transformation method, operational method, and the iteration method [21–23]. However, most of these methods are suitable only for special types of fractional differential equations, called linear with constant coefficients. Recently, there are some works dealing with the solutions of nonlinear fractional differential equations using techniques of nonlinear analysis such as: Adomian decomposition method (ADM) [36–42], variational-iteration method (VIM) [43–48], homotopy perturbation method (HPM) [49,50] and others. Adomian decomposition method [36–38] succeeded to solve accurately different types of fractional nonlinear differential equations by applying the Adomian polynomials. This method is applied to study many problems arising from applied sciences and engineering [39–42]. The variational-iteration method [43–46] was successfully employed to solve many types of linear, nonlinear and fractional differential equations that describe scientific and engineering problems [46–48]. As advantages over ADM, the VIM solves differential equations without using Adomian polynomials and has no linearization or perturbation for solving the nonlinear and fractional problems. The VIM principles for solving the differential equations are given in many papers, e.g. References 46–48. The VIM solution is provided as a convergent series, which may lead to exact solution for linear differential equations and to an efficient numerical solution for nonlinear and fractional differential equations. The series solution begins with a trial function that can be used as the solution of the linear term of the differential equation.

Our main motive here is to study the time fractional parameter effects on the propagation of solitary pressure waves through a fluid filled elastic tube. Therefore, beside what is considered in Reference 16, the derived nonlinear KdV equation is transformed using variational technique described by Agrawal [24–26] into the time fractional KdV (TFKdV) equation [45]. The TFKdV equation is solved by applying VIM [46–48] developed by He and the effect of the fractional order is studied.

This paper is organized as follows: The basic set of tube and fluid equations, which governs our system, is presented in section 2. In section 3, the KdV equation is derived by applying the reductive perturbation method [51]. Section 4 is devoted to derive and solve the time fractional KdV (TFKdV) equation using variational methods. Finally, some results and discussion are presented in section 5.

2. Basic equations of motion of the tube and fluid

A circular cylindrical long tube of un-deformed radius R_0, subjected to a uniform initial inner pressure P_0 is considered. A position vector $\underline{r}$ of a general point on the tube is described as [11,12]:

$$\underline{r} = [r_0 + u(z,t)]\underline{\hat{e}}_r + z\underline{\hat{e}}_z \tag{1a}$$

$$z = \lambda_z Z \tag{1b}$$

where r_0 is the radius of the tube after a finite static deformation, $u(z,t)$ is a finite dynamic time dependent deformation in the tube radius, $\underline{\hat{e}}_r$ and $\underline{\hat{e}}_z$ are the radial and axial unit vectors, respectively in the cylindrical polar coordinates. Z is the axial coordinate before the deformation, z is the axial coordinate after the static deformation and λ_z is the axial stretch ratio. The equation of motion of a small element of the tube's wall in the radial direction is given by [11,12]

$$\frac{R_0\rho_0}{\lambda_z}\frac{\partial^2 u(z,t)}{\partial t^2} = \frac{\partial}{\partial z}\left[\frac{R_0\mu}{\Lambda}\frac{\partial \sum(z,t)}{\partial \lambda_1}\frac{\partial u(z,t)}{\partial z}\right] - \frac{\mu}{\lambda_z}\frac{\partial \sum(z,t)}{\partial \lambda_2} + \frac{R_0\lambda_2}{H}P^*(z,t) \tag{2}$$

where ρ_0 and μ are the mass density and the shear modulus of the tube material, respectively. $\Sigma(z,t)$ is the strain energy density function, $P^*(z,t)$ is the fluid pressure at the final inner deformed tube radius r_f and H is the initial un-deformed tube thickness. r and z are the radial and axial cylindrical coordinates, respectively after both static and dynamic deformations and t is the time parameter. λ_1 and λ_2 are the axial and radial direction stretches, respectively and are represented by [11,12]

$$\lambda_1 = \lambda_z \Lambda \tag{3a}$$

$$\lambda_2 = \lambda_r + u(z,t)/R_0 \tag{3b}$$