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Nonlinear dynamic modeling of rotor system supported by angular contact ball bearings

Hong Wang [a], Qinkai Han [b,*], Daning Zhou [a]

[a] Beijing Institute of Control Engineering, Beijing 100000, China
[b] The State Key Laboratory of Tribology, Tsinghua University, Beijing 100084, China

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ABSTRACT

In current bearing dynamic models, the displacement coordinate relations are usually utilized to approximately obtain the contact deformations between the rolling element and raceways, and then the nonlinear restoring forces of the rolling bearing could be calculated accordingly. Although the calculation efficiency is relatively higher, the accuracy is lower as the contact deformations should be solved through iterative analysis. Thus, an improved nonlinear dynamic model is presented in this paper. Considering the preload condition, surface waviness, Hertz contact and elastohydrodynamic lubrication, load distribution analysis is solved iteratively to more accurately obtain the contact deformations and angles between the rolling balls and raceways. The bearing restoring forces are then obtained through iteratively solving the load distribution equations at every time step. Dynamic tests upon a typical rotor system supported by two angular contact ball bearings are conducted to verify the model. Through comparisons, the differences between the nonlinear dynamic model and current models are also pointed out. The effects of axial preload, rotor eccentricity and inner/outer waviness amplitudes on the dynamic response are discussed in detail.

1. Introduction

The rolling element bearing is the core supporting component of many rotating machinery. Its dynamic behavior plays a decisive role in the performance, operation reliability and service life of the entire equipment. It is well known that some key factors, including the Hertzian contacts, bearing radial clearance, surface waviness, preload condition and so on, would significantly affect the dynamic behaviors of rolling element bearings.

Thus, appropriately modeling one or more key factors of the bearing has been the focus of most current investigations. Jones [1] first analyzed the modeling of an elastically constrained ball and radial roller bearings, and proposed a general bearing model with five degrees of freedom considering gyroscopic moments and centrifugal forces of the balls. Tiwari et al. [2,3] proposed a widely used deep-groove ball bearing model taking into account both radial clearance and Hertzian contact characteristics. Considering the effect of radial internal clearance, Harsha et al. [4] presented a dynamic model for the deep-groove ball bearing based on the analytical mechanics method. Taking more factors into account, they subsequently set up an analytical model of the high-speed rotor supported on ball bearings [5]. Later, they also proposed a dynamic model of the

* Corresponding author.
E-mail address: hanqinkai@hotmail.com (Q. Han).

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Nomenclature

A_{in}, B_{in}, A_{out}, B_{out}, C_n various waviness amplitudes
$c_{\eta p}$ viscosity–pressure coefficient
$\mathbf{C}$ damping matrix of the rotor-bearing system
d_e pitch diameter of the bearing
d_b ball diameter
d_{ri}, d_{ro} diameters of inner and outer races
E material elastic modulus
E' effective elastic modulus
$f_{bL1}, f_{bL2}, f_{bL3}, f_{bL4}, f_{bL5}$ restoring forces of bearing L
$f_{bR1}, f_{bR2}, f_{bR3}, f_{bR4}, f_{bR5}$ restoring forces of bearing R
f_u, f_v, f_w, m^u, m^v preloads applied on the inner race
F_{cj}, M_{gj} centrifugal and gyroscopic forces due to the rotation and revolution of the ball
$\mathbf{f}_b$ resultant force vector of the outer race
$\mathbf{f}_p$ preload vector of the inner race
$\mathbf{F}_b$ nonlinear restoring force vector induced by the two bearings
$\mathbf{F}_e$ unbalanced mass excitation vector of the rotor-bearing system
$\mathbf{F}_g$ self-gravity vector of the rotor-bearing system
G dimensionless elastic modulus
$\mathbf{G}$ gyroscopic matrix of the rotor-bearing system
h_{ci}, h_{co} oil film thicknesses for the rolling ball contacting with the inner and outer raceways
IF_n characteristic frequencies of the inner waviness
k_{b1}, k_{b2}, k_{b3}, k_{b4}, k_{b5} five linear stiffness coefficients of the rotor system
k_{L1}, k_{L2}, k_{L12}, k_{R1}, k_{R2}, k_{R12} stiffness coefficients of the bearing supports
K_i,K_o contact stiffness coefficients between the ball and innner/outer races
$\mathbf{K}_n$ stiffness matrix of the rotor-bearing system
l_1,l_2 distances from rotor to bearing R and L
L_{ij}, l_{ij} distance from inner groove curvature center to the ball mass center before and after deflection
L_{oj}, l_{oj} distance from outer groove curvature center to the ball mass center before and after deflection
m_b,I_b mass and moment of inertia of the ball
m_L,m_R mass coefficients of the bearing supports
m_s, I_{sd}, I_{sp} mass, diametral and polar moments of inertia of the rotor
m_{su}, e_s unbalanced mass and eccentricity of the rotor
$\mathbf{M}$ mass matrix of the rotor-bearing system
n_{in}, n_{out}, n_{ba} harmonic orders of the waviness
N_b number of rolling ball
OF_n characteristic frequencies of the outer waviness
p_{ij}, p_{oj} circumferential waviness of inner and outer raceways
q_{ij}, q_{oj} axial waviness of inner and outer raceways
$\mathbf{q}$ system displacement vector
Q_{ij}, Q_{oj} contact forces between the ball and inner/outer races
Q_{yjL}, Q_{zjL}, $Q_{\theta xjL}$ reaction forces from the jth ball to the outer race of bearing L
Q_{yjR}, Q_{zjR}, $Q_{\theta xjR}$ reaction forces from the jth ball to the outer race of bearing R
r_{go} groove curvature radius of the outer race
r_{gi} groove curvature radius of the inner race
R_o distance from the rotating axis center of outer race to the groove curvature center
R_x equivalent radius along the ball rolling direction
t time
$\mathbf{T}_j$, $\mathbf{T}_L$, $\mathbf{T}_R$ transformation matrices
u, v, w, θ^u, θ^v five degrees of freedom of the inner race
u_{ent} equivalent rotating speed
u_L, v_L, u_R, v_R vibrational displacements of the bearing supports
u_s, v_s, w_s, θ_s^u, θ_s^v three translational and two angular displacements of the rotor
U dimensionless speed
$\mathbf{u}$ freedom vector of the inner race
w_{ij}, w_{oj} rolling ball's waviness
W dimensionless load
x–y–z rotating bearing coordinate
X–Y–Z fixed bearing coordinate
X_s–Y_s–Z_s rotor system coordinate
X_{sL}–Y_{sL}–Z_{sL} local coordinate for bearing L
X_{sR}–Y_{sR}–Z_{sR} local coordinate for bearing R
α_j angle between self-rotation axis and the z-axis
β_0 nominal contact angle
$\boldsymbol{\delta}_j$ displacement vector of the outer groove curvature center
δ_{ij}, δ_{oj} contact deflections between the ball and inner/outer races
$\Delta\mathbf{w}_j$ additional displacements due to the waviness
ζ_j, η_j, θ_j displacements of the outer curvature center corresponding the jth ball
η_0 lubricant viscosity at the atmospheric pressure and temperature
κ elliptical ratio
λ_{ij}, λ_{oj} friction coefficients between the ball and inner/outer races
ν Poisson's ratio
ξ, ϵ first and second kinds of elliptical integration
ς proportional damping coefficient
ϕ_0 initial location angle of the ball
ϕ_{cj} location angle of the jth ball
ψ_{in}, φ_{in}, ψ_{out}, φ_{out}, ϑ_{ba} phases of the waviness
ω_{cj} orbital speed of the jth ball
ω_o angular speed of outer race
ω_{sj} self-rotation angular speed of the jth ball
$\mathfrak{R}$ equivalent curvature radius

rotor-bearing system with the effect of cage run-out and finite number of balls [6]. Based upon the model presented by Tiwari and Harsha, further research was carried out. Tomovic et al. [7] performed the response analysis of a rigid rotor supported by unloaded ball bearings. Lioulios and Antoniadis [8] investigated the effect of fluctuations of the rotational