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Uncertainty quantification based on pillars of experiment, theory, and computation. Part I: Data analysis

I. Elishakoff [a,*], N. Sarlin [b]

[a] Department of Ocean and Mechanical Engineering, Florida Atlantic University, Boca Raton, FL 33431-0991, USA
[b] French Institute for Advanced Mechanics, Campus des Cézeaux Clermont-Ferrand, Rue Roche Genes, 63170 Aubière, France

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ABSTRACT

In this paper we provide a general methodology of analysis and design of systems involving uncertainties. Available experimental data is enclosed by some geometric figures (triangle, rectangle, ellipse, parallelogram, super ellipse) of minimum area. Then these areas are inflated resorting to the Chebyshev inequality in order to take into account the forecasted data. Next step consists in evaluating response of system when uncertainties are confined to one of the above five suitably inflated geometric figures. This step involves a combined theoretical and computational analysis. We evaluate the maximum response of the system subjected to variation of uncertain parameters in each hypothesized region. The results of triangular, interval, ellipsoidal, parallelogram, and super ellipsoidal calculi are compared with the view of identifying the region that leads to minimum of maximum response. That response is identified as a result of the suggested predictive inference. The methodology thus synthesizes probabilistic notion with each of the five calculi. Using the term "pillar" in the title was inspired by the News Release (2013) on according Honda Prize to J. Tinsley Oden, stating, among others, that "Dr. Oden refers to computational science as the "third pillar" of scientific inquiry, standing beside theoretical and experimental science. Computational science serves as a new paradigm for acquiring knowledge and informing decisions important to humankind". Analysis of systems with uncertainties necessitates employment of all three pillars.

The analysis is based on the assumption that that the five shapes are each different conservative estimates of the true bounding region. The smallest of the maximal displacements in x and y directions (for a 2D system) therefore provides the closest estimate of the true displacements based on the above assumption.

1. Introduction

Several approaches exist nowadays to approach the systems involving uncertain parameters. The central among these methodologies appears to be the probabilistic analysis. Drenick (1990) deals with the question on "how to accommodate uncertainties in the empirical and theoretical information which underlies… applications". He stresses: "That an accommodation is absolutely necessary that has been recognized for a long time in the natural sciences as well as in engineering, and traditional way of achieving it has been to equate uncertainty with probability. It is an idea that was,

* Corresponding author. Tel.: +1 561 297 2729; fax: +1 561 297 2885.
E-mail address: elishako@fau.edu (I. Elishakoff).

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without question, spectacularly successful". Later on additional approaches were developed, central among them being the methodology based on fuzzy sets. In recent decades interval analysis [15], ellipsoidal analysis, parallelepiped analysis [10], and super-ellipsoidal analysis [6] were introduced.

Systematic comparison of interval, ellipsoidal, and super-ellipsoidal approaches was recently conducted by Elishakoff and Elettro [7]. In the latter study authors postulated that the bounds of variation of uncertain variables are known. This paper introduces, in addition to the above three calculi, also the triangular and parallelogram calculi. It should be stressed that we assume that the five shapes are each different conservative estimates of the true bounding region. The smallest of the maximal displacements in x and y directions (for a two-dimensional system) therefore provide the closest estimates of the true displacements based on the above assumption.

The main contribution of this study is development of general methodology for analysis of systems exhibiting uncertainties. We show that hybrid use of the probabilistic and non-probabilistic approaches is needed in realistic situations with limited available experimental data. Specifically, probabilistic considerations are needed to be involved in order to determine the probable regions of variation of uncertain variables. Once such a region has been determined, non-probabilistic calculi comes into play. The proposed methodology can be viewed as a predictive inference for systems involving uncertainties.

2. Set of points

In this section, we consider the case of a specified hypothetical set of points that represent experimental data pertaining to a set of realizations of two-dimensional uncertain variable (X, Y). Rather than considering the generic case of data we "let the data speak" in terminology of Moore [14]. Indeed, in words of Sherlock Holmes, celebrated hero of Arthur Conan Doyle [8], "It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts". Velleman and Hoaglin [22] state: Data analysis starts with data. The data may come from a known structure, such as a designed experiment or a sample survey; or they may be serendipitous, collected with no particular analysis in mind or with some entirely different analysis in mind. Regardless of their source, we begin by examining the data, usually in graphical displays. At this stage we aim to get a general idea of any patterns while remaining open to unexpected features.

Following the latter advice, we illustrate the data in the Cartesian coordinate system Oxy with points (X_i, Y_i), i being the serial number of available points, totaling$n$. Specific set of the chosen points are shown in Fig. 1. The coordinates of experimental points are listed in Table 1.

In this particular set, $n = 8$. Our task is to bound these points by some closed figure, that will be used at the later stage for analysis of specific structures subjected to uncertainty represented by the above set of points. Naturally, there could be infinite amount for figures that bound a given set of experimental points. We will limit ourselves by five specific enclosing figures, namely (a) triangle, (b) rectangle, (c) parallelogram, (d) ellipse, (e) super ellipse. In order not to exaggerate uncertainty, it makes sense to find smallest area figures bounding the experimental data.

Let us introduce the convex hull of the above points. It is obtained via the numerical procedure *convhull* in Matlab®, bearing in mind that there could be only one convex hull per specified data. We denote the edge of this hull e_j, j being the serial number of edge in the convex hull. In our case we have the six-edged convex hull shown in Fig. 2.

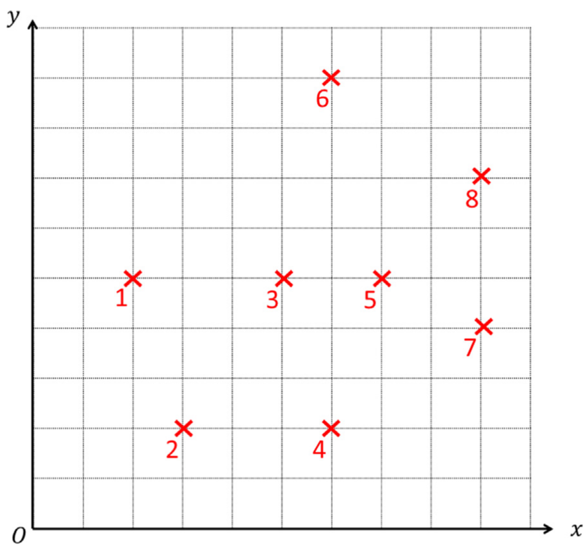

Fig. 1. The set of data points.