Mechanical Systems and Signal Processing 74 (2016) 54–72

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Uncertainty quantification based on pillars of experiment, theory, and computation. Part II: Theory and computation

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ARTICLE INFO

Article history:
Received 13 July 2014
Received in revised form
20 March 2015
Accepted 24 April 2015
Available online 17 June 2015

Keywords:
Interval analysis
Ellipsoidal analysis
Super ellipsoidal analysis
Most favorable response
Least favorable response

ABSTRACT

This paper deals with theoretical and computational aspects of different uncertainty calculi, introduced in Part I, specifically when the data is bounded by any of the following five figures: triangle; rectangle; parallelogram; ellipse or super ellipse. We consider elastic structures subjected to uncertainty, and evaluate the least favorable, maximum response and the most favorable, minimum response. Comparison is conducted between the treated uncertainty calculi with preference given to the one which predicts the least estimate for the favorable response. In considered elastic structures the solution or displacements is available analytically; in cases when analytical solution is absent purely numerical solution ought to be implemented. Such a case is now under development and will be published elsewhere.

1. Introduction

Analysis of uncertain structures can be conducted in various ways. The simplest possible methodology is interval analysis which treats uncertain parameters as intervals. There are numerous studies that deal with interval uncertainty. As example, we list papers by Dimarogonas [4], Köylüoğlu et al. [16], Qiu and Elishakoff [24], Muscolino et al. [21], Rao and Berke [25], McWilliams [19] and others.

Ellipsoidal modeling – usually referred to as convex modeling – was treated in several monographs as well as papers by Elishakoff [5], Pantelides and Ganzerli [23], Lombardi and Haftka [18], Jiang et al. [13] and others. Recently, two new convex models were pioneered: one deals with parallelepiped [14] whereas the other suggests application of super ellipsoids (Elishakoff and Beckel [7]). Comparison of interval, ellipsoidal and super ellipsoidal models was recently conducted by Elishakoff and Elettro [9].

In all above works the critical assumption was made that the interval, ellipsoid, parallelepiped, or super ellipsoid were provided to the analyst. These works did not address the issue forecasting the data and the need to inflate the uncertainty domains via Chebyshev inequality or other means, though combination of probabilistic and convex models was conducted in past two decades [8,12,22].

In this paper we compare *hybrid* uncertainty calculi. Each calculus is combined with probabilistic methodology to forecast the data that may appear in the future experiments. Systematic comparisons are conducted for various structures through evaluation of least favorable structural responses.

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http://dx.doi.org/10.1016/j.ymssp.2015.04.036
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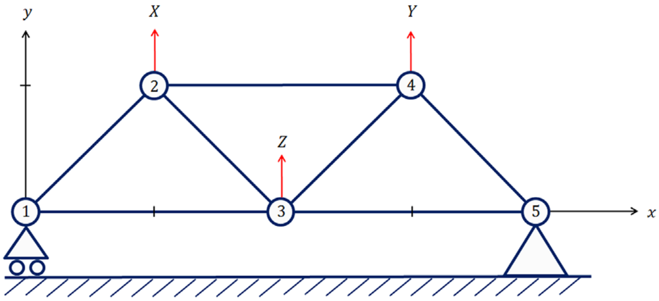

Fig. 1. A plane truss with three uncertain loads ($n=3$; $m=2$).

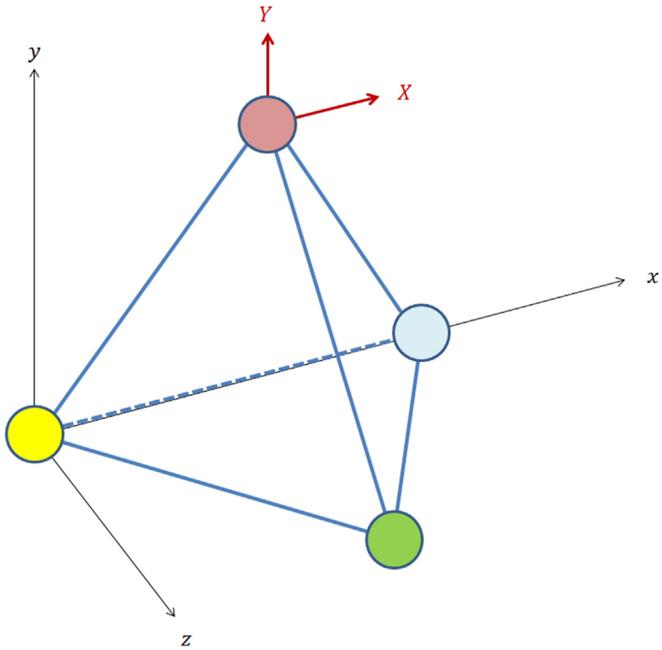

Fig. 2. A space truss with two loads ($n=2$; $m=3$).

2. Considering the data as loads

Let us identify the experimental data that was treated in Part I as loads acting on an elastic structure. With a two-dimensional data, we have a set of 2 uncertain loads; with a n-dimensional data we have a set of n uncertain loads. Let us consider a structure subjected to the uncertain loads. Let us denote by m the dimension of this structure. For instance, it is possible to study a single uncertain load on a tetrahedron: that means that a one-dimensional data is linked to a structure in space. The important thing is that in general there is no link between n and m. Moreover, it is possible to have n loads on a plane structure, with different ways and different points of application. Figs. 1 and 2 illustrate two different cases. In Fig. 1, a plane truss is subjected to three uncertain loads X, Y and Z whereas in Fig. 2 a space truss is under the action of two uncertain loads X and Y. Hereinafter, uncertain variables will be denoted with capital letters, whereas deterministic quantities by lower-case letters.

In the following, we limit ourselves with a particular case $n=m=2$. We focus on a two-dimensional problem in order to keep explanations as simple as possible. We keep considering the data (X, Y) as a set of points in a two-dimensional coordinate system, treating the coordinates X_i and Y_i of each point as loads, applied on a structure. Thus, the set of points corresponds to a set of different loads, leading to a resultant load denoted F_i for every point i from the data. System may also be subjected to a deterministic load z. It leads a resultant load depicted in Fig. 3:

$$\vec{F}_i = \vec{X}_i + \vec{Y}_i + \vec{Z}. \tag{1}$$

Let us introduce the following function, corresponding to an uncertain displacement resulting from loads X, Y and z:

$$U = pX + qY + rz \tag{2}$$

with p, q and r being constants depending on the structure.