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Identification of nonlinear normal modes of engineering structures under broadband forcing

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ABSTRACT

The objective of the present paper is to develop a two-step methodology integrating system identification and numerical continuation for the experimental extraction of nonlinear normal modes (NNMs) under broadband forcing. The first step processes acquired input and output data to derive an experimental state-space model of the structure. The second step converts this state-space model into a model in modal space from which NNMs are computed using shooting and pseudo-arclength continuation. The method is demonstrated using noisy synthetic data simulated on a cantilever beam with a hardening-softening nonlinearity at its free end.

1. Introduction

Experimental modal analysis of linear engineering structures is now well-established and mature [1]. It is routinely practiced in industry, in particular during on-ground certification of aircraft and spacecraft structures [2–4], using two specific approaches, namely phase resonance and phase separation methods. Phase resonance testing, also known as force appropriation, consists in exciting the normal modes of interest one at a time using a multipoint sine forcing at the corresponding natural frequency [5]. Conversely, in phase separation testing, several normal modes are excited simultaneously using either broadband or swept-sine forcing, and are subsequently identified using appropriate linear system identification techniques [6,7].

The existence of nonlinear behavior in dynamic testing is today a challenge for the structural engineer who is more and more frequently confronted with. In this context, the development of a nonlinear counterpart to experimental modal analysis would be extremely beneficial. An interesting approach to nonlinear modal testing is the so-called nonlinear resonant decay method introduced by Wright and co-workers [8,9]. In this approach, a burst of a sine wave is applied to the structure at the undamped natural frequency of a normal mode, and enables small groups of modes coupled by nonlinear forces to be excited. A nonlinear curve fitting in modal space is then carried out using the restoring force surface method. The identification of modes from multimodal nonlinear responses has also been attempted in the past few years. For that purpose, advanced signal processing techniques have been utilized, including the empirical mode decomposition [10–12], time-frequency analysis tools [13] and machine learning algorithms [14]. Multimodal identification relying on the synthesis of frequency response functions using individual mode contributions has been proposed in Refs. [15,16]. The difficulty with

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these approaches is the absence of superposition principle in nonlinear dynamics, preventing the response of a nonlinear system from being decomposed into the sum of different modal responses.

In the present study, we adopt the framework offered by the theory of nonlinear normal modes (NNMs) to perform experimental nonlinear modal analysis. The concept of normal modes was generalized to nonlinear systems by Rosenberg in the 1960s [17,18] and by Shaw and Pierre in the 1990s [19]. NNMs possess a clear conceptual relation with the classical linear normal modes (LNMs) of vibration, while they provide a solid mathematical tool for interpreting a wide class of nonlinear dynamic phenomena, see, *e.g.*, Refs. [20–23]. There now exist effective algorithms for their computation from mathematical models [24–27]. For instance, the NNMs of full-scale aircraft and spacecraft structures and of a turbine bladed disk were computed in Refs. [28–30], respectively.

A nonlinear phase resonance method exploiting the NNM concept was first proposed in Ref. [31], and was validated experimentally in Ref. [32]. Following the philosophy of force appropriation and relying on a nonlinear generalization of the phase lag quadrature criterion, this nonlinear phase resonance method excites the targeted NNMs one at a time using a multipoint, multiharmonic sine forcing. The energy-dependent frequency and modal curve of each NNM are then extracted directly from the experimental time series by virtue of the invariance principle of nonlinear oscillations. Applications of nonlinear phase resonance testing to moderately complex experimental structures were recently reported in the technical literature, in the case of a steel frame in Ref. [33] and of a circular perforated plate in Ref. [34].

The identification of NNMs from broadband data represents a distinct challenge in view of the absence of superposition principle in nonlinear dynamics. Indeed, the measured responses cannot merely be decomposed into a sum of individual NNM contributions. To address this challenge, the present paper develops a two-step methodology integrating system identification and numerical continuation for the experimental extraction of NNMs under broadband forcing. The first step processes acquired input and output data using the frequency-domain nonlinear subspace identification (FNSI) method [35] to derive an experimental state-space model of the structure. The second step converts this state-space model into a model in modal space from which NNMs are computed using shooting and pseudo-arclength continuation [25]. It should be noted that identification and continuation tools others than FNSI and pseudo-arclength may also qualify for the present framework. However, the two latter are adopted because of their accuracy and applicability to real-life structures.

The paper is organized as follows. The fundamental properties of NNMs defined as periodic solutions of the underlying undamped system are briefly reviewed in Section 2. The existing nonlinear phase resonance method introduced in Ref. [31] is also described. In Section 3, the two building blocks of the proposed NNM identification methodology, namely the FNSI method and the pseudo-arclength continuation algorithm, are presented. The methodology is demonstrated in Section 4 using noisy synthetic data simulated on a cantilever beam with a hardening-softening nonlinearity at its free end. Since it can be viewed as a nonlinear generalization of linear phase separation techniques, the proposed methodology is also compared in Section 5 with the previously-developed nonlinear phase resonance method. The conclusions of the study are finally summarized in Section 6.

2. Brief review of nonlinear normal modes (NNMs) and identification using phase resonance

In this work, an extension of Rosenberg's definition of a NNM is considered [23]. Specifically, a NNM is defined as a nonnecessarily synchronous, periodic motion of the undamped, unforced, n_p-degree-of-freedom (DOF) system

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)+\mathbf{f}(\mathbf{q}(t))=0, \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}\in\mathbb{R}^{n_p\times n_p}$ are the mass and linear stiffness matrices, respectively; $\mathbf{q}\in\mathbb{R}^{n_p}$ is the generalized displacement vector; $\mathbf{f}(\mathbf{q}(t))\in\mathbb{R}^{n_p}$ is the nonlinear restoring force vector encompassing elastic terms only. The definition of a NNM may appear to be restrictive in the case of nonconservative systems. However, as shown in Refs. [23,36], the topology of the underlying conservative NNMs of a system yields considerable insight into its damped dynamics.

Because a salient property of nonlinear systems is the frequency-energy dependence of their oscillations, the depiction of NNMs is conveniently realized in a frequency-energy plot (FEP). A NNM motion in a FEP is represented by a point associated with the fundamental frequency of the periodic motion, and with the total conserved energy accompanying the motion. A branch in a FEP details the complete frequency-energy dependence of the considered mode. Fig. 1 illustrates the FEP of the two-DOF system described by the following equations:

$$\begin{aligned}&\ddot{q}_1+(2q_1-q_2)+0.5q_1^3=0\\&\ddot{q}_2+(2q_2-q_1)=0.\end{aligned} \tag{2}$$

The plot features two branches corresponding to the in-phase and out-of-phase synchronous NNMs of the system. These fundamental NNMs are the direct nonlinear extension of the corresponding LNMs. The nonlinear modal parameters, *i.e.* the frequencies of oscillation and the modal curves, are found to depend markedly on the energy. In particular, the frequency of the two fundamental NNMs increases with the energy level, revealing the hardening characteristic of the cubic stiffness nonlinearity in the system.

Two essential properties of linear systems are preserved in the presence of nonlinearity. First, forced resonances of nonlinear systems occur in the neighborhood of NNMs [20]. Second, NNMs obey the invariance principle, which states that if the motion is initiated on one specific NNM, the remaining NNMs are quiescent for all time [19]. These two properties were exploited in Ref. [31] to develop a nonlinear phase resonance method. The procedure comprises two steps, as