Mechanical Systems and Signal Processing 74 (2016) 111–132

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

# Modal contributions and effects of spurious poles in nonlinear subspace identification

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ARTICLE INFO

*Article history:*
Received 2 July 2014
Received in revised form
18 March 2015
Accepted 4 May 2015
Available online 30 May 2015

*Keywords:*
Nonlinear identification
Stabilisation diagrams
Modal mass
Subspace methods

ABSTRACT

Stabilisation diagrams have become a standard tool in the linear system identification, due to the capability of reducing the user interaction during the parameter extraction process. Their use in the presence of nonlinearity was recently introduced and it was demonstrated to be effective even in presence of non-smooth nonlinearities and high modal density. However, some variability of the identification results was reported, in particular concerning the quantification of the nonlinear effects, because of the presence of spurious modes, due to an over-estimation of the system order.

In this paper the impact of spurious poles on the nonlinear subspace identification is investigated and some modal decoupling tools are introduced, which make it possible to identify modal contributions of physical poles on the nonlinear dynamics. An experimental identification is then conducted on a multi-degree-of-freedom system with a local nonlinearity and the significant improvements of the estimates obtained by the proposed approach are highlighted.



## 1. Introduction

It is well known that conventional linear estimators give contaminated results in presence of nonlinearities and the extraction of the underlying linear system properties hence becomes a difficult task. To solve this problem, in the last three decades several nonlinear identification methods have been developed, most of them being applicable to single-degree-of-freedom (SDOF) systems. More recently, multi-degree-of-freedom (MDOF) systems have been successfully dealt with [1]; however, only a limited number of nonlinear terms and degrees of freedom were included, due to the complexity of algorithms and required computational effort. To overcome these problems, the method called nonlinear subspace identification (NSI) has been proposed [2], by using the perspective of nonlinearities as unmeasured internal feedback forces [3]. This time domain method exploits the robustness and high numerical performances of algorithms, i.e., the basis of the stochastic subspace identification (SSI), successfully adopted in many linear applications [4–10]. A dual approach has been developed in the frequency domain, termed frequency-domain nonlinear subspace identification (FNSI) method [11], which allows to discriminate frequencies according to information content and signal-to-noise ratio (SNR), thus increasing the accuracy and reducing the computational burden.

It is a matter of fact that system identification results are improved by over-specifying the model order [6], computing system poles and then removing spurious poles. This is usually performed with the help of stabilisation diagrams,

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http://dx.doi.org/10.1016/j.ymssp.2015.05.008
0888-3270/

constructed by estimating poles with increasing model order and by plotting only those poles for which the relative difference in modal frequency, damping ratio and shape is below a user-defined value. In order to make stabilisation diagrams clearer, the modal transfer norm was proposed in [7] for a combined deterministic-stochastic subspace identification. Moreover, different techniques were introduced to reduce the user interaction: a clustering algorithm was proposed in [12], a component energy index was defined in [9] to estimate the model order and a hierarchical clustering algorithm was adopted in [10] for analyzing continuously collected data of a bridge.

The use of stabilisation diagrams in the presence of nonlinearity was first introduced in [13] and it was demonstrated to be effective for retrieving linear system parameters from nonlinear data generated by numerical experiments, even in presence of non-smooth nonlinearities, high modal density and high non-proportional damping. However, some variability of the identification results was reported, in particular concerning the quantification of the nonlinear effects.

The objective of the present paper is to investigate the role of spurious poles, to show how they affect the estimates of the nonlinear contributions to the system dynamics and how to improve the estimates. For this investigation, a modal decoupling procedure is introduced and the modal mass is computed for system poles. This procedure is illustrated by processing experimental measurements conducted on a scaled building connected to a metallic wire, which adds strong nonlinear effects. A new perspective is finally introduced, based on the identification of modal contributions due to physical poles on the nonlinear dynamics.

The paper is organised as follows. The theoretical background of the NSI is outlined in Section 2. This is followed by the description of modal decoupling tools (Section 3), which act in conjunction with stabilisation diagrams to remove spurious poles. A numerical validation is conducted in Section 4, where a system of low dimensions is studied. The experimental work is finally described in Section 5, where the parameter estimation is conducted both with low excitation level (linear identification) and high excitation level (nonlinear identification). The conclusions of the present study are summarised in Section 6.

## 2. Nonlinear subspace identification

Let us consider the equation of motion of a dynamical discrete system with $h$ degrees of freedom, carrying lumped nonlinear springs and dampers:

$$\mathbf{M}\ddot{\mathbf{z}}(t)+\mathbf{C}_v\dot{\mathbf{z}}(t)+\mathbf{K}\mathbf{z}(t)+\sum_{j=1}^{p}\mu_j\mathbf{L}_{nj}g_j(t)=\mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}_v$ and $\mathbf{K}$ are the mass, viscous damping and stiffness matrices respectively, $\mathbf{z}(t)$ is the generalised displacement vector and $\mathbf{f}(t)$ the generalised force vector, both of dimension $h$, at time $t$. The nonlinear term $\mathbf{N}[\mathbf{z}(t),\dot{\mathbf{z}}(t)]=\sum_{j=1}^{p}\mu_j\mathbf{L}_{nj}g_j(t)$ is expressed as the sum of $p$ components, each of them depending on the scalar nonlinear function $g_j(t)$, which indicates the class of the nonlinearity, through a location vector $\mathbf{L}_{nj}$, whose elements may assume the values 1, $-1$ or 0. By moving the nonlinear terms of Eq. (1) to the right hand side

$$\mathbf{M}\ddot{\mathbf{z}}(t)+\mathbf{C}_v\dot{\mathbf{z}}(t)+\mathbf{K}\mathbf{z}(t)=\mathbf{f}(t)-\sum_{j=1}^{p}\mu_j\mathbf{L}_{nj}g_j(t)=\mathbf{f}(t)-\mathbf{f}_{nl}(t) \tag{2}$$

the original system may be viewed as subjected to the external forces $\mathbf{f}(t)$ and the internal feedback forces caused by nonlinearities $\mathbf{f}_{nl}(t)$. This perspective, already chosen in [3] to develop the frequency domain method called nonlinear identification through feedback of the outputs (NIFO), is on the basis of the present time domain identification method, referred to as NSI [2]. In the case of measurements $\mathbf{y}$ involving displacements only, the state-space formulation of the equation of motion is

$$\begin{Bmatrix}\dot{\mathbf{z}}\\ \ddot{\mathbf{z}}\end{Bmatrix}=\begin{bmatrix}\mathbf{0}_{h\times h} & \mathbf{I}_{h\times h}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}_v\end{bmatrix}\begin{Bmatrix}\mathbf{z}\\ \dot{\mathbf{z}}\end{Bmatrix}+\begin{bmatrix}\mathbf{0}_{h\times h} & \mathbf{0}_{h\times 1} & \cdots & \mathbf{0}_{h\times 1}\\ \mathbf{M}^{-1} & \mathbf{M}^{-1}\mu_1\mathbf{L}_{n1} & \cdots & \mathbf{M}^{-1}\mu_p\mathbf{L}_{np}\end{bmatrix}\begin{bmatrix}\mathbf{f}(t)^T & -g_1(t) & \cdots & -g_p(t)\end{bmatrix}^T \tag{3}$$

$$\mathbf{y}=\begin{bmatrix}\mathbf{I}_{h\times h} & \mathbf{0}_{h\times h}\end{bmatrix}\begin{Bmatrix}\mathbf{z}\\ \dot{\mathbf{z}}\end{Bmatrix}+\begin{bmatrix}\mathbf{0}_{h\times h} & \mathbf{0}_{h\times 1} & \cdots & \mathbf{0}_{h\times 1}\end{bmatrix}\begin{bmatrix}\mathbf{f}(t)^T & -g_1(t) & \cdots & -g_p(t)\end{bmatrix}^T \tag{4}$$

corresponding to the state vector $\mathbf{x}=\begin{bmatrix}\mathbf{z}^{\mathbf{T}} & \dot{\mathbf{z}}^{\mathbf{T}}\end{bmatrix}^{\mathbf{T}}$ (superscript $T$ denotes transposition) and to the input vector $\mathbf{u}=\begin{bmatrix}\mathbf{f}(t)^T & -g_1(t) & \cdots & -g_p(t)\end{bmatrix}^T$ or, in a more compact form

$$\begin{aligned}\dot{\mathbf{x}}&=\mathbf{A}_c\mathbf{x}+\mathbf{B}_c\mathbf{u}\\ \mathbf{y}&=\mathbf{C}\mathbf{x}+\mathbf{D}\mathbf{u}\end{aligned} \tag{5}$$

This continuous model may be converted into the following discrete state-space model, assuming zero-order hold for the input $\mathbf{u}$ with sampling period $\Delta t$:

$$\mathbf{x}_{k+1}=\mathbf{A}\mathbf{x}_k+\mathbf{B}\mathbf{u}_k$$