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Analytical approximations of two and three dimensional time-fractional telegraphic equation by reduced differential transform method

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ABSTRACT

In this article, an analytical solution based on the series expansion method is proposed to solve the time-fractional telegraph equation (TFTE) in two and three dimensions using a recent and reliable semi-approximate method, namely the reduced differential transformation method (RDTM) subjected to the appropriate initial condition. Using RDTM, it is possible to find exact solution or a closed approximate solution of a differential equation. The accuracy, efficiency, and convergence of the method are demonstrated through the four numerical examples.

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1. Introduction

Several real phenomena emerging in engineering and science fields can be demonstrated successfully by developing models using the fractional calculus theory. Fractional differential theory has gained much more attention as the fractional order system response ultimately converges to the integer order equations. The applications of the fractional differentiation

for the mathematical modeling of real world physical problems such as the earthquake modeling, the traffic flow model with fractional derivatives, measurement of viscoelastic material properties, etc., have been widespread in this modern era. Before the nineteenth century, no analytical solution method was available for such type of equations even for the linear fractional differential equations. Recently, Keskin and Oturanc [1] developed the reduced differential transform

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method (RDTM) for the fractional differential equations and showed that RDTM is the easily useable semi analytical method and gives the exact solution for both the linear and nonlinear differential equations.

Let us assume that $u(x,y,t)$ and $i(x,y,t)$ denote the electric voltage and the current in a double conductor, then the time-fractional telegraphic equations (TFTEs) in the two dimension (2D) are given as

$$\left. \begin{aligned} \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}} + 2p \frac{\partial^\alpha u}{\partial t^\alpha} + q^2 u &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f_1(x, y, t), \\ \frac{\partial^{2\alpha} i}{\partial t^{2\alpha}} + 2p \frac{\partial^\alpha i}{\partial t^\alpha} + q^2 i &= \frac{\partial^2 i}{\partial x^2} + \frac{\partial^2 i}{\partial y^2} + f_2(x, y, t), \end{aligned} \right\}, (x, y, t) \in \Omega; p > 0, q > 0 \quad (1)$$

where $\Omega = [a, b] \times [c, d] \times [t > 0]$. The initial conditions are assumed to be

$$\left. \begin{aligned} u(x, y, 0) &= \phi_1(x, y), \\ u_t(x, y, 0) &= \phi_2(x, y), \\ i(x, y, 0) &= \chi_1(x, y), \\ i_t(x, y, 0) &= \chi_2(x, y), \end{aligned} \right\}, (x, y) \in \Omega \quad (2)$$

Similarly, the three dimensional (3D) time-fractional order telegraphic equation (TFTE) can be given as

$$\left. \begin{aligned} \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}} + 2p \frac{\partial^\alpha u}{\partial t^\alpha} + q^2 u &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + f_1(x, y, z, t), \\ \frac{\partial^{2\alpha} i}{\partial t^{2\alpha}} + 2p \frac{\partial^\alpha i}{\partial t^\alpha} + q^2 i &= \frac{\partial^2 i}{\partial x^2} + \frac{\partial^2 i}{\partial y^2} + \frac{\partial^2 i}{\partial z^2} + f_2(x, y, z, t), \end{aligned} \right\}, (x, y, z, t) \in \Omega; p > 0, q > 0 \quad (3)$$

where $\Omega = [a, b] \times [c, d] \times [e, f] \times [t > 0]$, with initial conditions

$$\left. \begin{aligned} u(x, y, z, 0) &= \xi_1(x, y, z), \\ u_t(x, y, z, 0) &= \xi_2(x, y, z), \\ i(x, y, z, 0) &= \psi_1(x, y, z), \\ i_t(x, y, z, 0) &= \psi_2(x, y, z) \end{aligned} \right\}, (x, y, z) \in \Omega \quad (4)$$

In Eqs. (1) and (3) p and q denote constants. For $p > 0, q = 0$, (1) and (3) represent time-fractional order damped wave equations in two and three dimensions respectively.

It has been observed that telegraph equation is more suitable than ordinary diffusion equation in modeling reaction diffusion. The hyperbolic partial differential equations model the vibrations of structures (e.g. machines, buildings and beams) and they are the basis for fundamental equations of atomic physics. The telegraph equation is an important equation for modeling several relevant problems in engineering and science such as wave propagation [2], random walk theory [3], signal analysis [4] etc. In recent years, from the literature it can be seen that much attention has been given to the development of analytical and numerical schemes for the one dimensional and two dimensional hyperbolic fractional and non-fractional TFTE [5–22]. To the best of our knowledge till now no one has applied the RDTM to solve the time-fractional order telegraphic equations in two and three dimensions.

In this paper, we propose an analytical scheme namely the reduced differential transformation method based on series solution method to find analytical solutions of the time-fractional telegraph equation (TFTE) in two and three dimensions. The accuracy and efficiency of the proposed method are demonstrated by the four test examples. The main advantage of the method is that it solves the telegraph

equation directly without using linearization, transformation, discretization or restrictive assumptions. Also, the RDTM scheme is very easy to implement for the multidimensional time-fractional order physical problems emerging in various fields of engineering and science.

2. Fractional calculus

In this section, we demonstrate some notations and definitions that will be used further in the study. Fractional calculus theory is almost more than two decades' old in the literature. Several definitions of fractional integrals and derivatives have been proposed but the first major contribution to give proper definition is due to Liouville as follows.

Definition 2.1. A real function $f(x), x > 0$ is said to be in the space $C_{\mu, \mu} \in \mathbb{R}$ if there exists a real number $q(>\mu)$, such that $f(x) = x^q g(x)$, where $g(x) \in C[0, \infty)$, and it is said to be in the space C_{μ}^m if $f^{(m)} \in C_{\mu, m} \in \mathbb{N}$.

Definition 2.2. For a function f , the Riemann–Liouville fractional integral operator [23] of order $\alpha \geq 0$, is defined as

$$\left\{ \begin{aligned} J^\alpha f(x) &= \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt, \alpha > 0, x > 0, \\ J^0 f(x) &= f(x) \end{aligned} \right. \quad (5)$$

The Riemann–Liouville derivative has certain disadvantages when trying to model real world problems with fractional differential equations. To overcome this discrepancy, Caputo and Mainardi [24] proposed a modified fractional differentiation operator D^α in his work on the theory of viscoelasticity. The Caputo fractional derivative allows the utilization of initial and boundary conditions involving integer order derivatives, which have clear physical interpretations.

Definition 2.3. The fractional derivative of f in the Caputo sense [25] can be defined as

$$D^\alpha f(x) = J^{m-\alpha} D^m f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-t)^{m-\alpha-1} f^{(m)}(t) dt, \quad (6)$$

for $m-1 < \alpha \leq m, m \in \mathbb{N}, x > 0, f \in C_{-1}^m$.

The fundamental basic properties of the Caputo fractional derivative are given as.

Lemma. If $m-1 < \alpha \leq m, m \in \mathbb{N}$ and $f \in C_{\mu}^m, \mu \geq -1$, then

$$\left\{ \begin{aligned} D^\alpha J^\alpha f(x) &= f(x), x > 0, \\ D^\alpha J^\alpha f(x) &= f(x) - \sum_{k=0}^m f^{(k)}(0^+) \frac{x^k}{k!}, x > 0, \end{aligned} \right. \quad (7)$$

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