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Resilient $\mathcal{H}_\infty$ filtering for discrete-time systems

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ABSTRACT

This paper studies the problem of resilient $\mathcal{H}_\infty$ filtering for discrete-time systems with norm-bounded uncertainties. The filter to be designed is assumed to have additive gain variations. Attention is focused on the design of a resilient $\mathcal{H}_\infty$ filter such that the filtering error system is asymptotically stable and has a prescribed $\mathcal{H}_\infty$ performance with respect to the uncertainty effects. Three different approaches are developed to design the $\mathcal{H}_\infty$ filter, and the design conditions are presented in terms of solutions to linear matrix inequalities (LMIs). Finally, an illustrative example is provided to show the effectiveness of the proposed approaches.

1. Introduction

Due to the fact that the state variables in control systems are not always available, the state estimation problem has received wide concern by many researchers [1–5]. In control theory and practice, an important approach to state estimation is $\mathcal{H}_\infty$ filtering [6,7], which can provide a guaranteed noise attenuation level. Compared with the traditional Kalman filtering, the $\mathcal{H}_\infty$ filtering approach does not require the exact knowledge of the statistics of the external noise signals and it is insensitive to the uncertainties both in the exogenous signal statistics and in dynamic models [8–16]. In the past few years, robust $\mathcal{H}_\infty$ filtering has become a hot topic in engineering literature and constitutes an integral part of state estimation research. By using various approaches, major developments in robust $\mathcal{H}_\infty$ filtering research have been proposed. In [17–19], the problem of $\mathcal{H}_\infty$ filtering for a class of linear uncertain systems is studied. Ref. [20] was concerned with the robust $\mathcal{H}_\infty$ filtering problem of a class of continuous-time-affine systems with implicit outputs. The parameter-dependent $\mathcal{H}_\infty$ filter design problem for output estimation in linear parameter varying (LPV) plants that include constant delays in the state was addressed in [21]. The problem of robust $\mathcal{H}_\infty$ filtering for impulsive stochastic systems under sampled measurements, which are subject to norm-bounded time-varying parameter uncertainties, was studied by [22]. In [23], the robust $\mathcal{H}_\infty$ filtering problem for a class of uncertain nonlinear time-delay stochastic systems was studied.

In the above results, the standard filters do not involve uncertainties and are assumed to be implemented exactly. However, in the actual engineering systems, the filters realized by microprocessors/microcontrollers do have some uncertainties due to limitation in available microprocessor/microcontroller memory, effects of finite word length of the digital processors and quantization of the A/D and D/A converters, and so on [24]. It implies that inaccuracies or uncertainties do occur in the implementation of designed filters. Thus, it is very interesting to study the design of $\mathcal{H}_\infty$ filters with respect to parametric gain perturbations (i.e., non-fragile, see [25]). In [26–28], the non-fragile $\mathcal{H}_\infty$ filtering problems for T-S fuzzy

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systems were studied. Ref. [29] was concerned with the problem of non-fragile $\mathcal{H}_\infty$ filter design for linear continuous-time systems, where the filter to be designed is assumed to include additive gain variations. Ref. [30] was concerned with the robust $\mathcal{H}_\infty$ filtering for a class of networked systems with distributed variable delays.

Although there has been a lot of research on robust and non-fragile $\mathcal{H}_\infty$ filtering, most results are developed for the case of a single uncertain problem (either plant parametric uncertainties or filter gain uncertainties). To date, very few works have addressed $\mathcal{H}_\infty$ filtering problem for systems with uncertainties in both plant and filter. Just in [31], the problem of resilient linear filtering for a class of linear continuous-time systems with norm-bounded uncertainties was investigated (where we use the phrase "resilient" to imply robustness with respect to both plant parametric uncertainties and filter matrices gain uncertainties). However, it should be noted that just two designed filter matrices are considered to have gain variations by [31], which implies that the presented results are limited for designing actual non-fragile filters.

In this paper, we will consider the problem of resilient $\mathcal{H}_\infty$ filtering problem for a class of discrete-time systems using norm-bounded uncertainties. Given an uncertain discrete-time system, our objective is to design a resilient $\mathcal{H}_\infty$ filter such that the filter error system is asymptotically stable and preserves a guaranteed $\mathcal{H}_\infty$ performance by considering uncertainties in both plant matrices and filter gain matrices. Different from the results [31] for resilient filtering, the proposed ones are toward all filter matrices with additive gain variations. Three different approaches are proposed to solve the $\mathcal{H}_\infty$ filtering problem by handling the uncertainties coupling. It is shown that the resilient $\mathcal{H}_\infty$ filter design conditions are formulated in terms of LMIs. An example is given to illustrate the effectiveness of the proposed design methods.

Notations: In symmetric block matrices, we use $*$ as an ellipsis for the terms that are introduced by symmetry. I is the identity matrix with appropriate dimension. $l_2[0, \infty)$ is the space of square-integrable vector functions over $[0, \infty)$. In addition, the notation $\text{He}(A) = A + A^T$ will also be used.

2. Problem statement and preliminaries

Consider a class of uncertain discrete-time system:

$$\begin{aligned} x(k+1) &= (A + \Delta_A(k))x(k) + Bw(k), \\ y(k) &= (C + \Delta_C(k))x(k) + Dw(k), \\ z(k) &= (L + \Delta_L(k))x(k) + Ew(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state variable, $y(k) \in \mathcal{R}^p$ is the measurement output, $z(k) \in \mathcal{R}^q$ is the signal to be estimated, $w(k) \in \mathcal{R}^v$ is the disturbance signal that is assumed to be the arbitrary signal in $l_2[0, \infty)$; $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times v}$, $C \in \mathcal{R}^{p\times n}$, $D \in \mathcal{R}^{p\times v}$, $L \in \mathcal{R}^{q\times n}$, and $E \in \mathcal{R}^{q\times v}$ are system matrices; $\Delta_A(k)$, $\Delta_C(k)$, and $\Delta_L(k)$ are unknown real norm-bounded matrices which represent time-varying parameter uncertainties.

In this paper, the uncertainties are considered to have the form [31]

$$\begin{pmatrix} \Delta_A(k) \\ \Delta_C(k) \\ \Delta_L(k) \end{pmatrix} = \begin{pmatrix} X_A \\ X_C \\ X_L \end{pmatrix} \Delta_x(k) Y, \tag{2}$$

where X_A, X_C, X_L, and Y are known real constant matrices with appropriate dimensions, $\Delta_x(k)$ is an unknown real matrix satisfying $\Delta_x^T(k)\Delta_x(k) \le I$.

To formulate the $\mathcal{H}_\infty$ filtering problem, consider a filter with gain variations of the following form:

$$\begin{aligned} x_f(k+1) &= (A_f + \Delta_{Af}(k))x_f(k) + (B_f + \Delta_{Bf}(k))y(k), \\ z_f(k) &= (C_f + \Delta_{Cf}(k))x_f(k) + (D_f + \Delta_{Df}(k))y(k), \end{aligned} \tag{3}$$

where $x_f(k) \in \mathcal{R}^n$ is the state of the filter, $z_f(k) \in \mathcal{R}^q$ is the estimation of $z(k)$, $A_f \in \mathcal{R}^{n\times n}$, $B_f \in \mathcal{R}^{n\times p}$, $C_f \in \mathcal{R}^{q\times n}$, and $D_f \in \mathcal{R}^{q\times p}$ are filter gain matrices to be determined, $\Delta_{Af}(k)$, $\Delta_{Bf}(k)$, $\Delta_{Cf}(k)$, and $\Delta_{Df}(k)$ represent the additive type of gain variations with the following form:

$$\begin{pmatrix} \Delta_{Af}(k) & \Delta_{Bf}(k) \\ \Delta_{Cf}(k) & \Delta_{Df}(k) \end{pmatrix} = \begin{pmatrix} X_{f1} \\ X_{f2} \end{pmatrix} \Delta_f(k) \begin{pmatrix} Y_{f1} & Y_{f2} \end{pmatrix}, \tag{4}$$

where X_{f1}, X_{f2}, Y_{f1}, and Y_{f2} are known real constant matrices with appropriate dimensions, $\Delta_f(k)$ is an unknown real matrix, and $\Delta_f^T(k)\Delta_f(k) \le I$.

Combining (1) and (3) yields the filtering error system:

$$\begin{aligned} x(k+1) &= (A + \Delta_A)x(k) + Bw(k), \\ x_f(k+1) &= (A_f + \Delta_{Af})x_f(k) + (B_f + \Delta_{Bf})((C + \Delta_C)x(k) + Dw(k)), \\ e(k) &= (L + \Delta_L)x(k) + Ew(k) - (C_f + \Delta_{Cf})x_f(k) - (D_f + \Delta_{Df})((C \\ &\quad + \Delta_C)x(k) + Dw(k)), \end{aligned} \tag{5}$$

where $e(k)$ is the filtering error as $e(k) = z(k) - z_f(k)$.

Then, the problem under consideration in this paper is:

Resilient $\mathcal{H}_\infty$ filtering problem: Design an $\mathcal{H}_\infty$ filter in the form of (3) such that the following specifications are satisfied for the filtering error system (5) with any uncertainties satisfying $\Delta_x^T(k)\Delta_x(k) \le I$ and $\Delta_f^T(k)\Delta_f(k) \le I$: (i) is asymptotically stable when $w(k) = 0$; (ii) has a prescribed level γ of $\mathcal{H}_\infty$ noise attenuation, i.e. under the initial condition $x(0) = x_f(0) = 0$, $\sum_{k=0}^{\infty} e^T(k)e(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k)$ is satisfied for any nonzero $w(k) \in l_2[0, \infty)$.

Remark 1. In a previous study [31], the problem of resilient Kalman filtering for a class of linear continuous-time systems with norm-bounded uncertainties was investigated, in which just two designed filter matrices were considered to have gain variations, i.e., A_f and B_f. It is easy to see that when $Y_{f2} = 0$ in (4), the $\mathcal{H}_\infty$ filter presented in this paper reduces to one in [31].

In next section, we will study design conditions for the $\mathcal{H}_\infty$ filter in the form of (3), that is, to determine the filter gain matrices in (3) such that the filtering error system (5) is quadratically stable with the prescribed $\mathcal{H}_\infty$ performances.

3. Resilient $\mathcal{H}_\infty$ filter design

For the resilient $\mathcal{H}_\infty$ filtering problem of the filtering error system (5), one difficulty is that two kinds of uncertainties exist in the system simultaneously, and it