Signal Processing 127 (2016) 191–205

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Polytopic H_∞ filter design and relaxation for nonlinear systems via tensor product technique

Xiangdong Liu [a,b], Yin Yu [a,b], Zhen Li [a,b,*], Herbert H.C. Iu [c]

[a] School of Automation, Beijing Institute of Technology, Beijing 100081, China
[b] Key Laboratory for Intelligent Control & Decision on Complex Systems, Beijing Institute of Technology, Beijing 100081, China
[c] School of Electrical, Electronic and Computer Engineering, University of Western Australia, Crawley, WA 6009, Australia

ARTICLE INFO

Article history:
Received 12 July 2015
Received in revised form
24 February 2016
Accepted 3 March 2016
Available online 11 March 2016

Keywords:
Nonlinear system
Polytopic H_∞ filter
Filter relaxation
Tensor product model transformation

ABSTRACT

This paper focuses on the intensive relaxation of the conservativeness inherently existing in the primitive stage of the polytopic H_∞ filter design for general nonlinear systems. The considered conservativeness intrinsically stems from the polytopic representation of the nonlinear system. Therefore, it is difficult to obtain a suitable polytopic representation in the case of the special requirement on small enough conservativeness. In this paper, the vertices of the polytopic representation and representation error are especially incorporated into the filter solution together, such that the deeply-relaxed filter can be achieved with the trade-off between the tightness of the vertex polytope and the representation error. Concretely, based on the TP model transformation and an ad hoc rectification, a well-adjusted polytopic representation is acquired for the nonlinear system by two independent steps, where the first step guarantees the satisfactory representation error, while the second step further assures the tightness of the vertex polytope. Finally, numerical simulations are provided to demonstrate the effectiveness and feasibility of the method in terms of both the polytopic filter relaxation and the filter performance.

1. Introduction

The nonlinear filtering is of significant importance in the field of control and signal processing as most physical systems are nonlinear in nature [1], methods of which have been widely applied in target tracking, navigation and attitude estimation and so on [2–4].

Among all, the extended Kalman filter (EKF) is a well-known option, where the linear Kalman filter is directly applied to the first-order local linearization of the nonlinear system [2]. As a counterpart, the so-called sigma-point filter, such as the unscented Kalman filter (UKF) and cubature Kalman filter (CKF), were proposed to reach a higher-order linearization precision [2,5]. These filters get effective with Gaussian assumptions, which will be degraded under non-Gaussian or unknown disturbances and greatly limit their application to practical systems. In such cases, a promising way is to conduct the nonlinear filtering under the H_∞ setting. The H_∞ filter has no requirement on the prior-knowledge of the noise inputs but only assumes that they have bounded energy. Moreover, the H_∞ approach provides both a guaranteed attenuation from the noise to the estimation error and robustness against the system uncertainty. In the past years, much effort has been made to develop H_∞ variants of these mentioned filters, such as [6,7] and the references therein. For example, a high-degree cubature H_∞ filter was proposed in [6]. And in [7], the traditional H_∞ filter, which has the similar form with the EKF, was presented. Furthermore, the nonlinear H_∞ filtering problem has also been studied by directly solving the nonlinear Hamilton–Jacobi inequalities (HJIs). But the HJIs are generally tough to handle [8,9]. The practical applications of the H_∞ filters can be found in many areas [3,6,10].

On the other hand, most filters mentioned above are essentially on the basis of the local linearization except the HJIs-based one. Therefore, this kind of filters has the potential divergence problem caused by the large linearization error in the circumstance of strong nonlinearity and the costly online computation induced by the Jacobian matrices' update or the sigma points' propagation [4,8]. In recent years, the acquisition of a global alternative for the nonlinear filtering problem has been explored upon the global linearization scheme [4,8,11]. Overall, these methods comprise two stages: at first, globally linearize the nonlinear system, i.e., represent the nonlinear system with polytopic models, and secondly, transform the nonlinear filter design problem into a convex optimization problem subjected to linear matrix inequalities (LMIs). These global solutions were termed as global linearization

* Corresponding author at: School of Automation, Beijing Institute of Technology, Beijing 100081, China.
E-mail addresses: xdliu@bit.edu.cn (X. Liu), yuyin@bit.edu.cn (Y. Yu), zhenli@bit.edu.cn (Z. Li), herbert.iu@uwa.edu.au (H.H.C. Iu).

http://dx.doi.org/10.1016/j.sigpro.2016.03.005
0165-1684/

design method in [8] and polytopic linearization differential inclusions (PLDIs)-based method in [4,11], respectively. The widely-cited book [12] by Boyd provides more information about the global linearization, the PLDIs and the polytopic systems. The terminology of polytopic filter design is adopted in this paper since the filter will be designed as polytopic form.

Compared with the local linearization scheme and the HJIs-based filtering approaches, the global solution shows dramatic advantages including small linearization error, high real-time efficiency and ease to design with the LMIs. It has been demonstrated to be a feasible and effective nonlinear filtering methodology in real applications. For instance, it was used to design robust H_2/H_∞ filter for the nonlinear stochastic systems from the practical design perspective [8]. It was also applied to develop a real-time efficient spacecraft attitude estimator under limited computing resources in [4]. A mixed H_2/H_∞ filter was designed for the nonlinear systems with non-Gaussian noise in [11], where the classical filtering application, in which a vertically falling ball was tracked using a range measure radar, was illustrated. However, the concern remains that it is likely to suffer from much potential conservativeness. The resultant conservativeness can be classified into two classes: the foremost is inherent to the polytopic representation in the first stage and the subsequent is brought by the LMIs' formulation in the second stage.

The transformation to LMIs in the second stage also exists in the filtering design for other systems, such as the linear systems subjected to convex-bounded uncertain parameters, the TS fuzzy systems and the polynomial systems [9,13–27]. The approaches to alleviate this kind of conservatism have been extensively investigated. There are two widely recognized sources of this type conservativeness. One is the limited choices of the Lyapunov function, which could be relieved by using parameter-dependent Lyapunov functions, such as [13–15,17–22,24]. The other one is the reduction of the degree of freedom accompanied by the transformation from nonlinear parameter-dependent matrix inequalities to LMIs, which could be alleviated by new relaxed stability theorems with introducing extra slack variables or increasing the number of LMIs, such as [9,17,19–21,27] and also the homogeneous polynomially parameter-dependent (HPPD) methods, such as [16,21–24].

However, the conservativeness in the first stage inherently stems from the polytopic representation of the nonlinear system, so that it is completely different from that in the second stage and exclusive to the polytopic filter design of the nonlinear system. Consequently, it gets much more serious than the later one since it comes into being in the primitive stage and improper representation often leads to the infeasibility of the subsequent LMIs. So, the polytopic representation has to be carefully determined. Nevertheless, it is generally difficult to acquire a polytopic representation, especially a feasible and conservativeness-reduced polytopic representation. To the best of the authors' knowledge, it has not been considered for the polytopic filter design of the general nonlinear systems.

Motivated by the aforementioned investigation, this paper aims to exploring a well-adjusted polytopic representation of a nonlinear system such that the polytopic H_∞ filter of the nonlinear system is relaxed. The TP model transformation is utilized as the initial calculation of a well-adjusted polytopic representation. This transformation, firstly introduced by Baranyi [28], is capable of transforming a linear parameter-varying (LPV) system into TP model form. The resultant TP model is still a polytopic one as long as the interpolation functions is normalized towards the convex combination, which is one of the main steps of the TP model transformation. Essentially, it is an effective numerical method based on the high-order singular value decomposition (HOSVD) [29] and has been widely used to design the straightforward and numerically tractable controller for various practical systems, such as aeroelastic system [30], translational oscillations with an eccentric rotational proof mass actuator (TORA) system [31], Rossler's chaotic system and Lorentz's chaotic system [28]. In this paper, on the one hand, in order to deeply relax the conservatism, both the vertices of the polytopic representation and the representation error are incorporated into the filtering solution that is characterized by LMIs, on the basis of a less accurate polytopic representation of the nonlinear system. On the other hand, a rectification procedure is specifically developed to rectify the initial result obtained by the TP model transformation. By doing so, a well-adjusted polytopic representation is determined by two separate steps. The first step acquires a representation candidate with a satisfactory representation error and the second step guarantees a tight enough vertex polytope, where there is no need to worry about the increase of the representation error. As a consequence, the H_∞ filter can be well relaxed.

The remaining of this paper is arranged as follows. In Section 2, the polytopic filter relaxation problem of nonlinear systems is formulated. In Section 3, the filtering convex optimization problem constrained by LMIs is established in an setting of H_∞ measure. Section 4 presents the calculation and rectification of the polytopic representation based on the TP model transformation. In Section 5, the simulation is demonstrated for the verification. Finally, Section 6 concludes the paper.

Throughout this paper, $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. I represents identity matrix with appropriate dimensions and $I_{n\times n}$ represents identity matrix with size $n \times n$. For a real matrix A, A^T denotes its transpose, and $*$ denotes the symmetric sub-block of a matrix. $(a_{ij})_{m\times n}$ stands for a $m \times n$ matrix with a_{ij} being its (i,j)-th entry. $\|\cdot\|$ is the Euclidean norm for a vector or the Frobenius norm for a matrix. $\mathcal{L}^2[0,\ T,\ \mathbb{R}^n] \triangleq \{x(t)\colon [0,\ T] \to \mathbb{R}^n | \int_0^T \| x(t)\|^2\, dt < \infty\}$ is the set of square integrable vector functions. For $x(t) \in \mathcal{L}^2[0,\ T,\ \mathbb{R}^n]$, $\| x \|_2 \triangleq (\int_0^T \| x(t)\|^2\, dt)^{\frac{1}{2}}$ is the $\mathcal{L}_2$ norm of x. The notation Γ_N is the N-dimensional unit simplex and $A_\lambda \triangleq \sum_{i=1}^N \lambda_i A_i$, $\lambda \in \Gamma_N$ denotes a polytopic matrix determined by the vertices A_i, $i = 1, 2, \ldots, N$. The boldface capital letters $\boldsymbol{A}$, $\boldsymbol{B}$, … denote high dimensional matrices, which are termed as tensor.

2. Problem statement

Consider the following nonlinear system:

$$\dot{x}(t) = f(x(t)) + g_1(x(t))v(t),$$
$$y(t) = h_1(x(t)) + g_2(x(t))v(t), \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the system state, $y(t) \in \mathbb{R}^{n_y}$ is the measurement output, $v(t) \in \mathbb{R}^{n_v}$ is an unknown disturbance with limited energy, i.e., $v(t) \in \mathcal{L}^2[0,\ T,\ \mathbb{R}^{n_v}]$. Assume that $f(x)$ and $h_1(x)$ are once continuously differentiable with respect to each component of x, $g_1(x)$ and $g_2(x)$ are continuous, and x_e is an equilibrium of the system.

In the following, we first manifest that the nonlinear system can be globally recast as a polytopic representation, and a less accurate polytopic representation is especially preferable, from the perspective of the conservativeness reduction. Then, a polytopic filter is introduced using the polytopic representation. Furthermore, the polytopic filter relaxation problem, which is concerned within this paper, is proposed.

Proposition 1. *Consider the nonlinear system (1) and let $\theta(t) = x(t) - x_e$ and $\eta(t) = y(t) - h_1(x_e)$. Given a compact domain $\mathbb{D} \subset \mathbb{R}^{n_x}$ and $0 \in \mathbb{D}$, then $\theta(t)$ and $\eta(t)$ can be exactly represented in $\mathbb{D}$, by the following polytopic form*: