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Estimation of parametric convergence bounds for Volterra series expansion of nonlinear systems

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ABSTRACT

The convergence bound for Volterra series expansion of nonlinear systems is investigated with a novel parametric approach in this study. To this aim, two fundamental concepts — parametric bound of convergence (PBoC) and parametric convergence margin (PCM) are proposed, which are related to the conditions, under which a given NARX model can be approximated by a convergent Volterra series, in terms of system characteristic parameters including model parameters (of interest), input magnitude, and frequency. The estimation of the PBoC and PCM is given in the frequency domain, which is expressed in terms of these characteristic parameters, and does not require iterative calculations. The results provide a fundamental basis for nonlinear analysis and design using Volterra series based methods, and also present a significant insight into understanding nonlinear influence (super/sub harmonics and modulation) with respect to model parameters and input magnitude. Several examples are given to illustrate the effectiveness of the results.

1. Introduction

Many nonlinear systems can be identified into a Nonlinear AutoRegressive with eXogenous inputs (NARX) model [1–3], which includes several commonly-used nonlinear models as special cases. The NARX model actually provides a generic and convenient platform for analysis and design of nonlinear systems in practice. Given a parametric nonlinear model, several methods are available in the literature for nonlinear analysis and design [4–7]. For those systems described by NARX models, the nonlinear analysis and design can also be done in the frequency domain using the concept of Generalized Frequency Response Function (GFRF) [8]. The latter is defined as the multidimensional Fourier transform of Volterra kernels of the Volterra series expansion of the original nonlinear system. It is known that the input–output relationship of a considerably large class of nonlinear systems allows a Volterra series expansion [9–11,17,18]. For this reason, the Volterra series has been widely used in the literature for various nonlinear analysis and design [7,11–18].

Usually, the Volterra series expansion of the input–output relationship of a given NARX model should be confined into a specific region in order to ensure an accurate approximation of the original nonlinear dynamics. There are several results in the literature attempting to provide a convergence criterion under which a convergent Volterra series expansion exists. But some of these results are only applicable to specific nonlinear systems such as duffing oscillators [19–21], and others are either too general to apply for a specific parametric nonlinear model [9–12,17,18,22], or too conservative [23] or obviously over-estimated [19–21]. Noticeably, all existing results focus only on a convergent criterion in terms of input magnitude.

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Nomenclature

ω	frequency variable
Ω	the output frequency
W_∞	the whole output frequency range
U	input magnitude
$c_{p,q}(k_1,\dots,k_{p+q})$	model parameters with nonlinear degree p in terms of output and nonlinear degree q in terms of input
$h_n(\tau_1,\dots,\tau_n)$	nth order Volterra kernel
$L_n(j\omega_1,\dots,j\omega_n)$	the function of the linear model parameters $c_{1,0}(k_1)$
$H_n(j\omega_1,\dots,j\omega_n)$	nth order Generalized Frequency Response Function (GFRF)
$\overline{H}_n(j\omega_1,\dots,j\omega_n)$	upper bound of nth order GFRF
$C(p,q)$	nonnegative function of model parameters $c_{p,q}(k_1,\dots,k_{p+q})$
M_p	the maximum nonlinear degree in terms of output
$\underline{L}(\omega)$	lower bound of function $\|L_n(j\omega_1,\dots,j\omega_n)\|$
$\overline{Y}_{\Omega=k\omega}(U)$	upper bound of nonlinear output spectrum at Ω with input magnitude U
$x=\overline{Y}(U)_\omega$	upper bound of the nonlinear output spectrum involves the whole output frequency range with input magnitude U, which is denoted as x
C_n^m	take m combinations in the given n elements
Γ	an indicator for convergence margin
PBoC	parametric bound of convergence
PCM	parametric convergence margin

In practical analysis and design of a nonlinear system, a fundamental problem could be: in what parameter ranges (in terms of the input magnitude or model parameters for a given input or at a given frequency) can the system have a convergent Volterra series expansion? More specifically, the task could only involve designing a particular model parameter. The question could be: under what range can the parameter take freely its value such that the system is always valid for a convergent Volterra series expansion? These practical questions are clearly key issues (for any nonlinear analysis and design using the Volterra series theory), but still not well addressed.

In this study, parametric convergent bounds in terms of some characteristic parameters including model parameters, magnitude bound of the first order GFRF (relating to linear model parameters), input magnitude, and frequency variables are studied for the NARX model in order to have a convergent Volterra series expansion. Firstly the analytical representation of the relationship between the upper bound of nonlinear output spectrum and characteristic parameters is presented. Then, the concept of parametric bound of convergence (PBoC) is discussed and the estimation of the PBoC for the NARX model is proposed, which clearly indicates in what parametric ranges a given nonlinear system has a convergent Volterra series expansion. Finally, a new convergence concept with respect to Volterra series expansion – parametric convergence margin (PCM) – is proposed, which can give a quantitative evaluation of the convergence margin in terms of any characteristic parameters for a given nonlinear system before the Volterra series expansion diverges. These new concepts and results should provide a significant basis and useful guidance for nonlinear analysis and design using the Volterra series based theory and methods [24–28], and can also present a new insight into understanding of nonlinear influence (e.g., super/sub-harmonic response) incurred by different characteristic parameters. Examples are given to illustrate these theoretical results.

2. Frequency response functions of nonlinear systems

2.1. The NARX model

Consider nonlinear systems described by the NARX model:

$$y(k)=\sum_{m=1}^{M} y_m(k) \tag{1a}$$

$$y_m(k)=\sum_{p=0}^{m}\sum_{(k_1,\cdots,k_m)} c_{p,m-p}(k_1,\cdots,k_m)\prod_{i=1}^{p} y(k-k_i)\prod_{i=p+1}^{m} u(k-k_i) \tag{1b}$$

where M is the maximum nonlinear degree in terms of $y(k)$ and $u(k)$, p is the nonlinear degree in terms of $y(k)$, and $m-p$ is the nonlinear degree in terms of $u(k)$ which is denoted later by $q=m-p$. $(k_1,\dots,k_m)$ denotes all the combinations of nonlinear terms in terms of input and output, which can be expressed as $(k_1,\dots,k_m)\in\mho_m=\{(k_1,\dots,k_m)|1\le k_i\le K,$ $p\le k_1+\cdots+k_p\le pK, q\le k_{p+1}+\cdots+k_m\le qK\}$, where K is the maximum order of derivative, and $c_{p,m-p}(k_1,\cdots,k_m)$ is the corresponding coefficient of term $\prod_{i=1}^{p} y(t-k_i)\prod_{i=p+1}^{m} u(t-k_i)$. The NARX model (1) above can be approximated by a Volterra series expansion as [9–11,18]:

$$y(k)=\sum_{n=1}^{N}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_n(\tau_1,\dots,\tau_n)\prod_{i=1}^{n} u(k-\tau_i)\,d\tau_i \tag{2}$$

where N is the truncation order, and $h_n(\tau_1,\dots,\tau_n)$ is the nth order Volterra kernel.