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Complex eigenvector scaling from mass perturbations

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ABSTRACT

This paper presents an approach to normalize experimentally extracted complex eigenvectors so that their outer product gives transfer function residues. The approach, an implementation of the mass perturbation strategy, is exact for arbitrary perturbation magnitudes and number of sensors when the modal space is complete and is robust against modal truncation. It is shown that improvements over a sensitivity solution are significant when the relation between the eigenvalue and the perturbation magnitude is strongly nonlinear.

1. Introduction

Results from output only modal identification cannot be used to establish input–output relations because the information to properly scale the mode shapes is unavailable. Since a number of applications in experimental dynamics and diagnostics are based on input–output maps, procedures to determine the modal scaling constants for output only settings are of practical interest. A strategy that has received notable attention computes these constants by equating the eigenvalue sensitivity (with respect to a given perturbation) to an experimental estimate, the simplest being a finite difference approximation from the results of two tests [1]. In theory the type of perturbation is arbitrary but in practice the addition of masses is typically easiest to implement and has thus been most commonly considered [2–7].

Scaling constants estimated using the mass perturbation scheme are random variables with a bias and a variance that depend on the perturbation magnitude and on the specifics of how the information is used to extract the constants. For conciseness in the discussion we introduce some notation from the outset, namely, we parameterize the mass perturbation as $\Delta M = \beta M_1$, where β is a scalar and note that as $\beta \to 0$ the information vanishes and thus $\sigma^2 \to \infty$, where σ^2 is the variance of the scaling constant estimate. It is evident, therefore, that to realize a reasonable variance sufficiently large perturbations are needed. As β gets larger, however, the bias of a finite difference estimate of the sensitivity tends to increase because the relation between the eigenvalue λ and the perturbation magnitude β is generally nonlinear. Two different paths to minimize the bias issue can be pursued. The first one is to select distributions of the perturbation that minimize nonlinearity in $\lambda(\beta)$ (or in a predetermined function of this function) [3,4] and the second is to formulate the problem in such a way that the solution depends on the total changes in the eigenvalues, instead of the derivatives. Belonging to the first alternative one can mention the use of distributions M_1 that more or less mimic the actual mass, as these lead to quasi-linear relations between the inverse of the (undamped) eigenvalue and the perturbation magnitude. The limiting case, however,

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being redundant since in this instance the modes have to be measured "everywhere" and knowledge of the mass distribution allows normalization using the orthogonality relationship. Optimization of the positioning of a few masses to reduce nonlinearity in (some) of the $\lambda(\beta)$ relations is analytically feasible but this requires use of a model and is thus incompatible with the data-driven constraints that apply to the present problem.

Formulations that work with the total changes in the eigenvalues (instead of the derivatives) do not impose constraints on the number, or on the spatial distribution of the perturbation, and this is the framework pursued in this paper. At first glance it may appear that working with total changes would remove the perturbation magnitude bias altogether but this is not generally the case because finite perturbations lead to modal coupling and bias enters the problem through the modally truncated space. A formulation for the scaling constants based on total eigenvalue changes, however, can offer significant reductions gains over a sensitivity scheme because the bias in this case (as shall be shown) is only weakly dependent on β. The first formulation that used total changes to estimate the scaling constants traced the nonlinear $\lambda(\beta)$ relation using sensitivities for a set of assumed constants and obtained the solution by minimizing discrepancy between predictions and identified results [2]. Albeit straightforward conceptually, the outlined optimization framework is computationally cumbersome compared to a direct solution and, perhaps for this reason, has not received much attention. Methods that offer direct solutions include the Projection Approach (PA) [5], derived for the normal mode model on the premise that the perturbed eigenvectors lay on the basis of the unperturbed modes and the Receptance Based Normalization technique (RBN) [7], where the constants are obtained from an over-determined linear system of equations.

Implicit throughout the previous discussions is the fact that the modes that are to be normalized are the real modes of the normal mode model. While the normal mode model is sufficiently accurate in the large majority of cases, there are instances, typically due to the existence of a set of modes with small eigenvalue gaps (see Appendix A) where irreducible eigenvector complexity is significant and the more general first order modal model must be used to avoid undue error [8]. This paper presents an extension of the RBN scheme to the normalization of the complex eigenvectors of the first order formulation. The paper presents the theory, examines performance in a stochastic setting and clarifies the conditions were the gains with respect to a sensitivity solution are important. The contrast between the complex form, designated as CRBN, and the approach that applies in the case of normal modes is highlighted in Appendix B for convenience.

2. Preliminaries

Consider an arbitrarily viscously damped linear time invariant model, accepting finite dimensionality the equations of dynamic equilibrium can be written as

$$M\ddot{q}(t)+C\dot{q}(t)+Kq(t)=f(t) \tag{1}$$

where $M, C, K \in R^{N\times N}$ are the mass damping and stiffness matrices, N is the number of degree of freedom and $f(t)$ are the applied loads. Taking a Laplace transform writes

$$[Ms^2+Cs+K]q(s)=f(s) \tag{2}$$

Solutions are possible for $f(s)=0$ if the matrix in the parenthesis is rank deficient. The values of s for which this matrix loses rank are the poles, or complex eigenvalues, λ, and the vectors in the null space are the latent vectors ψ [9,10]. It is customary to refer to the latent vectors as complex eigenvectors, or simply eigenvector. For any value of s, other than a pole, the matrix in Eq. (2) can be inverted and one has

$$q(s)=G(s)f(s) \tag{3}$$

where $G(s)$ is known as the Receptance. The Receptance can be expressed as [11]

$$G(s)=\sum_{j=1}^{2N}\frac{\psi_j\psi_j^T\rho_j}{s-\lambda_j} \tag{4}$$

taking

$$\kappa_j=\sqrt{\rho_j} \tag{5}$$

and defining the normalized complex eigenvectors as

$$\varphi_j=\kappa_j\psi_j \tag{6}$$

the Receptance writes

$$G(s)=\sum_{j=1}^{2N}\frac{\varphi_j\varphi_j^T}{s-\lambda_j} \tag{7}$$

For the eigenvectors in Eq. (7) one has, among other relations [10]

$$\Phi^TC\Phi+\Lambda\Phi^TM\Phi+\Phi^TM\Phi\Lambda=I \tag{8}$$