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A local correspondence principle for mode shapes in structural dynamics

Rune Brincker [a,*], Anders Skafte [a], Manuel López-Aenlle [b], Aldo Sestieri [c], Walter D'Ambrogio [d], Alfonso Canteli [b]

[a] Department of Engineering, Aarhus University, Denmark
[b] Universidad de Oviedo, Spain
[c] Università La Sapienza, Roma, Italy
[d] Università de L'Aquila, Italy

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ABSTRACT

It is well known that experimentally obtained mode shapes can be smoothed by using a linear combination of mode shapes from a finite element (FE) model. This is known from the theory of structural modification (SM) and from the system equivalent reduction expansion process (SEREP). Using this approach the set of FE mode shapes to be included in the smoothing must be chosen a priori and the quality of the smoothing and a subsequent mode shape expansion depend significantly on this choice. The present paper provides a solution to the problem of choosing which mode shapes are the most important for the smoothing and how many of the mode shapes should be included in order to obtain an optimal solution. It is shown based on the classical sensitivity theory that for each experimental mode shape, a mode shape cluster can be defined for the mode shapes of the FE model that defines an optimal choice for the smoothing set. The sequence of FE mode shapes to be included in this mode shape cluster is prescribed by a simple principle denoted the principle of local correspondence (LC) the name referring to the fact that an experimentally obtained mode shape should not be considered as corresponding to a single FE mode shape, but rather as corresponding to the mentioned mode shape cluster. A test case for a steel plate is considered where the experimentally obtained mode shapes are smoothed using SEREP (using a fixed set of mode shapes) and using the LC principle, and it is shown that the LC principle secures a high quality of the smoothing whereas the SEREP provides results that are strongly dependent upon the actual choice of the included FE mode shapes and on the degrees of freedom included in the fitting set.

1. Introduction

The classical way to relate experimentally obtained mode shapes and finite element (FE) mode shapes is to compare the modes one-by-one estimating the mode shape correlation between the experimental and the corresponding FE mode shape. The classical way to deal with a bad correlation is to perform an updation of the FE model.

In this paper we will consider another way of estimating the correlation between an experiment and a model, that is the correlation between an experimentally obtained mode shape and a linear combination of FE modes, and we will show how

* Corresponding author. Tel.: +39 4521474754.
E-mail address: rub@science.au.dk (R. Brincker).

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Nomenclature

$\mathbf{M},\mathbf{K}$	mass and stiffness matrix
$\mathbf{T}$	transformation matrix for SEREP and SM
$\mathbf{P}$	transformation matrix for the LC principle
λ	eigenvalue
ω	angular frequency (rad/s)
ω_m	angular frequency of mode number m
f	frequency (Hz)
ς	damping ratio
m	modal mass
$\mathbf{a}$	experimental mode shapes (un-scaled)
$\mathbf{b}$	(FE) model mode shapes (un-scaled)
$\boldsymbol{\beta}$	(FE) model mode shapes (mass scaled)

this linear combination of FE modes can be used to smooth the experimental mode shape and for expansion of the experimental mode shapes to unknown degrees of freedom (DOFs).

The classical way to calculate the correlation between a mode shape **a** from an experiment and a mode shape **b** from a FE model is to calculate the MAC value, Allemang et al. [1]

$$MAC(i,j) = \frac{|\mathbf{a}_i^H \mathbf{b}_j|^2}{(\mathbf{a}_i^H \mathbf{a}_i)(\mathbf{b}_i^H \mathbf{b}_i)} \tag{1}$$

where the super index "H" means hermitian (transpose complex conjugate). The MAC value is also equal to the cosine squared of the generalized angle θ between the two vectors **a** and **b**, and when the MAC is close to unity it might be more descriptive to use the generalized angle estimated as

$$\theta = \arccos(\sqrt{MAC(i,j)}) \tag{2}$$

as a measure of how much the two vectors deviate from full (unity) correlation.

If an experimentally obtained mode shape is to be compared with one of two closely spaced modes $\mathbf{b}_1$, $\mathbf{b}_2$ in the FE model, then it is known that in this case the correlation with the individual mode shapes is often low. For this case it has been suggested by D'Ambrogio and Fregolent [2] to correlate the experimental mode shape with a linear combination of the two mode shapes in the model.

Using the traditional definition of the MAC as given by Eq. (1), but now taking the vector from the model as an unknown linear combination of the two considered mode shapes and maximizing the higher order MAC denoted S2MAC, the following expression is obtained:

$$S2MAC = \max_{\alpha,\beta} \frac{|\mathbf{a}^H(\alpha \mathbf{b}_1 + \beta \mathbf{b}_2)|^2}{\mathbf{a}^H \mathbf{a} (\alpha \mathbf{b}_1 + \beta \mathbf{b}_2)^H (\alpha \mathbf{b}_1 + \beta \mathbf{b}_2)} \tag{3}$$

In D'Ambrogio and Fregolent [2] it is shown that the analytical solution to Eq. (3) for unit length real mode shapes is given by

$$S2MAC = \frac{(\mathbf{a}^T \mathbf{b}_1)^2 - 2(\mathbf{a}^T \mathbf{b}_1)(\mathbf{b}_1^T \mathbf{b}_2)(\mathbf{a}^T \mathbf{b}_2) + (\mathbf{a}^T \mathbf{b}_2)^2}{1 - (\mathbf{b}_1^T \mathbf{b}_2)^2} \tag{4}$$

The rationale for the present paper is that sometimes it is a good idea to relate an experimental mode shape not just to a single mode shape from a FE model as we do when we calculate the MAC as given by Eq. (1), or in case of closely spaced modes to two modes from a FE like given by Eq. (4), but in general to a subspace of mode shapes from a FE model.

The problem is closely related to the expansion theorem that states that any deformation of a mechanical system can be expressed as linear combination of the mode shapes. Considering a truncated series of FE mode shapes introduces the FE subspace and the problem then remains of how to choose the FE subspace so that we get the best approximation of the experimental mode shape.

The advantages of using such an approach is mainly that some of the noise on the experimental mode shape estimate can be removed to obtain a smoothed version of the experimental mode shape and that the smoothed version can be expanded to all DOFs of the FE model. The smoothing nearly always makes sense because of measurement errors introduced by the sensors, but the expansion might be even more valuable using traditional discrete sensors like accelerometers and of less value in cases where photometric high resolution techniques are used.

First in Section 2 we will introduce the idea of projecting the experimental mode shape on a fixed subspace and we will note that it is the same as using SEREP. Then based on classical sensitivity analysis in Section 3 we will formulate the LC principle that states that a perturbed mode shape can always be expressed as a linear combination of a limited number of unperturbed mode shapes around (in terms of frequency) the corresponding unperturbed mode shape. Finally in Section 4 we will discuss how many unperturbed mode shapes (here the FE mode shapes) that should be included in order to obtain the best smoothing, and finally we will illustrate how the LC principle works on an experimental test on a plate in free–free conditions.