Mechanical Systems and Signal Processing 45 (2014) 207–224

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Subspace-based damage detection under changes in the ambient excitation statistics ☆

Michael Döhler [a,*,1], Laurent Mevel [b], Falk Hille [c]

[a] Northeastern University, Department of Civil and Environmental Engineering, Center for Digital Signal Processing, 360 Huntington Avenue, Boston, MA 02115, USA
[b] Inria, Centre Rennes—Bretagne Atlantique, Campus de Beaulieu, 35042 Rennes, France
[c] BAM Federal Institute for Materials Research and Testing, Safety of Structures Department, 12200 Berlin, Germany

ARTICLE INFO

Article history:
Received 13 July 2012
Received in revised form
3 June 2013
Accepted 26 October 2013
Available online 16 November 2013

Keywords:
Damage detection
Structural vibration monitoring
Ambient excitation
Subspace methods
Hypothesis testing

ABSTRACT

In the last ten years, monitoring the integrity of the civil infrastructure has been an active research topic, including in connected areas as automatic control. It is common practice to perform damage detection by detecting changes in the modal parameters between a reference state and the current (possibly damaged) state from measured vibration data. Subspace methods enjoy some popularity in structural engineering, where large model orders have to be considered. In the context of detecting changes in the structural properties and the modal parameters linked to them, a subspace-based fault detection residual has been recently proposed and applied successfully, where the estimation of the modal parameters in the possibly damaged state is avoided. However, most works assume that the unmeasured ambient excitation properties during measurements of the structure in the reference and possibly damaged condition stay constant, which is hardly satisfied by any application. This paper addresses the problem of robustness of such fault detection methods. It is explained why current algorithms from literature fail when the excitation covariance changes and how they can be modified. Then, an efficient and fast subspace-based damage detection test is derived that is robust to changes in the excitation covariance but also to numerical instabilities that can arise easily in the computations. Three numerical applications show the efficiency of the new approach to better detect and separate different levels of damage even using a relatively low sample length.

1. Introduction

The fault detection problem is related to the detection and diagnosis of changes in the eigenstructure of a linear dynamical system in many applications. An important example is structural vibration monitoring, where damages of civil, mechanical or aeronautical structures lead to a change in the eigenstructure of the underlying mechanical system and thus in the modal parameters (natural frequencies, damping ratios and mode shapes). The excitation of these systems is ambient and mostly unmeasured. Vibration-based damage detection methods have been developed extensively in the last 30 years. In [2], an introduction to vibration-based damage identification is given. An overview of damage identification methods and strategies can be found in [3–5].

☆ A preliminary version of this paper was presented at the 18th IFAC World Congress, August 28–September 2, 2011, Milan, Italy [1].

* Corresponding author. Tel.: +49 8104 1701; fax +49 8104 1727.
E-mail addresses: michael.doehler@inria.fr, michael.doehler@bam.de (M. Döhler), laurent.mevel@inria.fr (L. Mevel), falk.hille@bam.de (F. Hille).
[1] Present address: BAM Federal Institute for Materials Research and Testing, Safety of Structures Department, 12200 Berlin, Germany.

0888-3270/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ymssp.2013.10.023

Nomenclature

$\mathbb{R}$, $\mathbb{C}$	sets of real and complex numbers
$\mathbf{E}$	expectation of a random variable
$\mathcal{N}(m,\Sigma)$	multivariate Gaussian distribution
$\otimes$	Kronecker product
vec	column stacking vectorization operator
$\dagger$	Moore–Penrose pseudoinverse
$\widehat{(\cdot)}$	estimate of a quantity
A, C	system matrices
x_k, y_k	states and outputs of system
v_k, w_k	state and output noise
Q	excitation covariance, cov(v_k)
$\tilde{Q}$	excitation covariance in tested state
$\mathcal{Q}$, $\mathcal{R}$	matrices in QR decomposition
R_i	output covariance at lag i
$\mathcal{H}_{p+1,q}$	Hankel matrix of output covariances R_i
$\mathcal{O}_{p+1}$, $\mathcal{C}_q$	observability, controllability matrix
θ_0, θ	system parameter in reference and current state
$\delta\theta$	change in system parameter
$\mathbf{H}_0$, $\mathbf{H}_1$	null and alternative hypotheses
$S(\theta_0)$	left null space of $\mathcal{O}_{p+1}$ in reference state
U_1	matrix of principal singular vectors of $\mathcal{H}_{p+1,q}$
ζ_N, ξ_N	residual vectors computed on N data points
$\mathcal{J}_\zeta$, $\mathcal{J}_\xi$	asymptotic sensitivity of residual vector
Σ_ζ, Σ_ξ	asymptotic covariance of residual vector
χ^2_N, γ^2_N	asymptotically χ^2-distributed test variables
$\overline{(\cdot)}$	variable for non-parametric test variant

Often, damage detection is done by estimating the modal parameters of a structure in a possibly damaged state and comparing them to estimates from a reference condition, e.g. by using control charts [6–8]. In this context, especially the natural frequencies are used for a comparison, as they can be reliably identified. However, the automated estimation of modal parameters from a system identification method and matching them from measurements of different states of the structure for their comparison might require an extensive preprocessing step. Other methods avoid the system identification step in the possibly damaged state and use e.g. outlier analysis for damage detection [9], Kalman filter innovations [10,11] or other data-driven features that are sensitive to changes in the modal parameters. The methods [12–14] considered in this paper compare a model obtained in the reference state to data from the possibly damaged state using a subspace-based residual function and a χ^2-test built on it for a hypothesis test, without actually estimating the modal parameters in the tested, possibly damaged states. This approach provides a complete statistical framework for analyzing a damage residual for both reference and possibly damaged structures. The asymptotic probability distribution of the considered damage residual for both structural states is given and a statistical hypothesis test to detect damage in the structure has been proposed.

While the modal parameters of a structure are not afflicted by a change in the ambient excitation statistics, damage detection tests that avoid the system identification step and use directly the measured vibration data are possibly perturbed by changes in the excitation covariance. We assume stationary excitation during the measurement of one data set, while the excitation covariance may change *between* measurements. The subspace-based damage detection algorithm [12] takes into account the statistical properties of the ambient excitation in its formulation, but it is shown that it is not robust to a change in the excitation covariance. In this paper, it is shown how this test can be corrected easily by recomputing the residual covariance for every tested data set at the expense of an increase in computational burden and loss of accuracy.

In parallel, a similar subspace-based damage residual that is robust to a changing excitation covariance has been proposed in [15]. Detecting a change in the structural properties by means of such a residual is possible by a hypothesis testing procedure involving the norm of the residual pondered by a precise estimate of its covariance as explained in [12]. In this paper, the statistical framework of [12] is extended to the robust residual. The statistical properties of the robust residual are analyzed and the corresponding hypothesis test is derived. A fast and numerically stable computation of the residual covariance and the damage detection test is proposed. The efficiency of this approach and its robustness to changes in the excitation covariance are demonstrated on three numerical examples and compared to the damage detection tests from [12–14].

This paper is organized as follows. After presenting the basic principles of statistical subspace-based fault detection from [12,13] in Section 2, the impact of a changing ambient excitation covariance between measurements on the damage detection test is discussed in Section 3. A damage detection test that is robust to a changing excitation covariance is proposed in Section 4, where a robust residual function and its corresponding hypothesis test are derived. In Section 5, simplified non-parametric versions of the presented damage detection tests are derived. In Section 6, a fast and numerically robust computation of the residual covariance and the damage detection test is proposed and in Section 7 an algorithmic summary of the new robust damage detection test is given. In Section 8, numerical results of the proposed algorithms are presented.

2. Background of statistical subspace-based fault detection

In this section, the basic concepts of stochastic subspace-based fault detection from [12,13] are presented.