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Projection-based robust adaptive beamforming with quadratic constraint

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ABSTRACT

A new robust adaptive beamforming technique is proposed in this study to address performance degradation of adaptive beamforming methods in the presence of steering vector mismatch. Actual steering vector of desired signal is estimated by solving a convex optimization problem with the objective constructed by minimizing the sum of estimated desired steering vector projections onto noise eigenvectors. The beamformer performs well at high signal-to-noise ratio (SNR) with the orthogonality between presumed desired steering vector and mismatch vector as a single constraint. Feasibility and necessity of adding an additional quadratic constraint are verified through detailed performance analysis with random matrix theory, improving the performance at low SNR. The parameter determination approach is provided to allow the proposed beamformer to function properly in practical situations. Both the theoretical analysis and simulation results demonstrate the proposed method is robust against any steering vector mismatch.

1. Introduction

Adaptive beamforming has been widely applied in wireless communications, radar, astronomy, medical imaging, and other fields [1,2]. Conventional adaptive beamforming methods suppress interference effectively and maximize the output signal-to-interference-plus-noise ratio (SINR) with exact knowledge of desired signal steering vector and interference-plus-noise covariance matrix (INCM). Precise knowledge is difficult or impossible to obtain in practical situations and the presence of steering vector mismatch, including look direction error and antenna array geometry perturbations, results in severe performance degradation of adaptive beamformers, thus many approaches have been proposed for improving robustness against steering vector mismatch.

Diagonal loading [3] is a prevalent robust adaptive technique. However, the selection method for optimal diagonal loading factor remains ambiguous. Robust Capon beamformer [4] can be regarded as a diagonal loading method with performance proven to be equivalent to the worst-case performance optimization method [5]. Through maximizing beamformer output power, an advanced beamformer [6] is proposed to estimate desired signal steering vector based on angular sector where desired signal is located. A method utilizing an orthogonal interference-plus-noise subspace projection matrix is introduced in [7] and the dimension of estimated interference-plus-noise subspace is determined by an energy percentage parameter while no clear selection guideline is provided. Jiang et al. proposed a class of beamformers exploiting non-Gaussianity of the signals [8–10], where minimum dispersion criterion is adopted in place of conventional minimum variance criterion.

INCM reconstruction [11,12] has been recently introduced as a new beamformer design principle. The INCM is reconstructed based on the Capon spatial spectrum in [11]

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and on the interference direction-of-arrival (DOA) and power estimation of interference and noise in [12]. Both algorithms perform perfectly for ideal antenna array; however, the presence of array perturbations (i.e., mutual coupling, sensor gain, and phase and sensor position errors) leads to severe performance degradation rendering the two approaches not suitable for practical situations. The conclusion is also applicable for beamformer in [13] where both interference-plus-noise covariance matrix and desired signal-plus-noise covariance matrix are constructed.

A new robust adaptive beamforming method is proposed in this study that offers robustness against steering vector mismatch with the beamformer formulated as an optimization problem. The frequently utilized objective function which maximizes beamformer output power is replaced in the study by minimizing the sum of estimated desired steering vector projections onto noise eigenvectors. Constraints of the optimization problem are constructed with imprecise knowledge of antenna array geometry and angular sector where the desired signal is located. The optimization problem is convex and can be easily solved utilizing convex optimization theory.

The paper is organized as follows: Section 2 introduces background material with the new beamformer proposed and analyzed in Section 3. Section 4 compares performance of the proposed method with existing beamformers and Section 5 draws the conclusions.

2. The signal model

A linear antenna array with M omni-directional sensors is considered. The output of narrowband beamformer at time instant k is given by

$$y(k)=\mathbf{w}^H\mathbf{x}(k), \tag{1}$$

where $\mathbf{w}$ is the weight vector of the antenna array and $(\cdot)^H$ represents the Hermitian transpose. The received signal $\mathbf{x}(k)$ can be written as

$$\mathbf{x}(k)=\mathbf{s}(k)+\mathbf{i}(k)+\mathbf{n}(k)=s_0(k)\mathbf{a}_0+\mathbf{i}(k)+\mathbf{n}(k), \tag{2}$$

where $\mathbf{a}_0$ is the actual desired signal steering vector, $s_0(k)$ is waveform of desired signal, $\mathbf{s}(k)$, $\mathbf{i}(k)$, and $\mathbf{n}(k)$ represent desired signal, interference, and noise, respectively. Desired signal and interference are assumed to be statistically independent of each other. Additive noise is modeled as a spatially and temporally independent complex zero-mean Gaussian process with identical variances in each array sensor.

The theoretical covariance matrix $\mathbf{R}_x$ is expressed as

$$\mathbf{R}_x=E[\mathbf{x}(k)\mathbf{x}^H(k)]=\sigma_0^2\mathbf{a}_0\mathbf{a}_0^H+\mathbf{R}_{i+n}, \tag{3}$$

where $E\{\cdot\}$ denotes statistical expectation, σ_0^2 is the desired signal power and $\mathbf{R}_{i+n}=E\{(\mathbf{i}(k)+\mathbf{n}(k))(\mathbf{i}(k)+\mathbf{n}(k))^H\}$ is interference-plus-noise covariance matrix. Output SINR corresponding to specific weight vector $\mathbf{w}$ can be formulated as

$$\mathrm{SINR}=\frac{\sigma_0^2|\mathbf{w}^H\mathbf{a}_0|^2}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}}. \tag{4}$$

and the maximization of (4) results in the minimum variance distortionless response (MVDR) problem

$$\min_{\mathbf{w}}\ \mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}$$
$$\text{s.t. } \mathbf{w}^H\mathbf{a}_0=1. \tag{5}$$

Solution of optimization problem (5) is

$$\mathbf{w}_{\mathrm{opt}}=\frac{\mathbf{R}_{i+n}^{-1}\mathbf{a}_0}{\mathbf{a}_0^H\mathbf{R}_{i+n}^{-1}\mathbf{a}_0}. \tag{6}$$

$\mathbf{R}_{i+n}$ and $\mathbf{a}_0$ cannot be obtained in practical applications, therefore, sample covariance matrix $\hat{\mathbf{R}}=1/N\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}^H(k)$ with N training snapshots and presumed steering vector $\bar{\mathbf{a}}$ are exploited, respectively. (6) is transformed to sample matrix inversion (SMI) adaptive beamformer

$$\mathbf{w}_{\mathrm{SMI}}=\frac{\hat{\mathbf{R}}^{-1}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H\hat{\mathbf{R}}^{-1}\bar{\mathbf{a}}}. \tag{7}$$

Limited training snapshots and imperfect knowledge of desired steering vector are known to result in performance degradation of the beamformer (7).

3. The proposed algorithm

Performance of the beamformer will improve with more accurate estimation of $\mathbf{R}_x$ or $\mathbf{a}_0$. A new robust adaptive beamforming method to estimate actual steering vector of desired signal is proposed in this section.

3.1. Basic beamformer

The theoretical covariance matrix $\mathbf{R}_x$ and sample covariance matrix $\hat{\mathbf{R}}$ can be decomposed as

$$\mathbf{R}_x=\mathbf{U}_S\boldsymbol{\Sigma}_S\mathbf{U}_S^H+\mathbf{U}_N\boldsymbol{\Sigma}_N\mathbf{U}_N^H=\sum_{i=0}^{M-1}\gamma_i\mathbf{e}_i\mathbf{e}_i^H,$$
$$\hat{\mathbf{R}}=\sum_{i=0}^{M-1}\hat{\gamma}_i\hat{\mathbf{e}}_i\hat{\mathbf{e}}_i^H, \tag{8}$$

where $\gamma_0\ge\cdots\ge\gamma_D=\cdots=\gamma_{M-1}$ are eigenvalues of $\mathbf{R}_x$ and $\mathbf{e}_i$ fwhereare corresponding eigenvectors, $\hat{\gamma}_i$ are eigenvalues of $\hat{\mathbf{R}}$ in descending order and $\hat{\mathbf{e}}_i$ are corresponding eigenvectors, D denotes the number of signal impinging on the array, $\mathbf{U}_S=[\mathbf{e}_0,\cdots,\mathbf{e}_{D-1}]$ spans the signal subspace, $\mathbf{U}_N=[\mathbf{e}_D,\cdots,\mathbf{e}_{M-1}]$ spans the noise subspace, $\boldsymbol{\Sigma}_S$ and $\boldsymbol{\Sigma}_N$ represent eigenvalue matrices of signal subspace and noise subspace, respectively.

Eigenvectors corresponding to large projections of desired steering vector $\mathbf{a}_0$ onto the eigenvector $\hat{\mathbf{e}}_i$ can be applied to construct space considered to be the same as signal subspace [14]. Eigenvectors corresponding to the small projections can be utilized to construct the new noise subspace. Under the assumption that the mismatch between $\mathbf{a}_0$ and $\bar{\mathbf{a}}$ is not too large, $\bar{\mathbf{a}}$ can be utilized to replace $\mathbf{a}_0$ with the above conclusions approximately remaining. The value of projections is given by

$$p(i)=\left|\hat{\mathbf{e}}_i^H\bar{\mathbf{a}}\right|\quad i=0,1,\cdots,M-1. \tag{9}$$