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Widely linear minimum dispersion beamforming for sub-Gaussian noncircular signals

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ABSTRACT

We propose two widely linear minimum dispersion based beamforming for sub-Gaussian noncircular signals: widely linear minimum dispersion distortionless response (WL-MDDR) and widely linear quadratically constrained minimum dispersion (WL-QCMD). In the nonideal conditions, taking full advantages of noncircularity and sub-Gaussian properties of signals, the proposed algorithms are shown to achieve good performance even in high signal-to-noise ratio and process more signals than the number of sensors. The WL-MDDR beamformer is designed when the information about the steering vector and noncircularity coefficient of the desired signal is precise, while the WL-QCMD beamformer is robust against arbitrary errors in the steering vector and noncircularity coefficient. Numerical simulations verify the effectiveness of the proposed algorithms.

1. Introduction

Beamforming is a fundamental technique in array signal processing, and widely used in radar, sonar and wireless communications [1]. Conventional beamforming techniques, such as minimum variance distortionless response (MVDR), are often based on the minimum variance (MV) criterion [2,3], which is statistically optimal only when the true covariance matrix and steering vector of the desired signal are available. However, due to the nonideal conditions, such as signal direction error, uncalibrated array, the finite number of snapshots and so on, this conclusion is not always true in practical applications. It is only suitable for a Gaussian signal. The reason is that the first- and second-order statistics of a Gaussian distribution contain all necessary statistical information.

But many real-world signals are non-Gaussian [4,5], which can be classified into sub-Gaussian and super-Gaussian based on the kurtosis [6,7]. For a random stationary signal $s(k)$ with zeros-mean, the kurtosis is defined as $\kappa(s(k)) = \left(\mathrm{E}\left[|s(k)|^4\right] - 2\mathrm{E}\left[|s(k)|^2\right]^2 - \left|\mathrm{E}\left[s(k)^2\right]\right|^2 \right) \Big/ \sigma_s^4$ [8], where σ_s^2 denotes the variance of $s(k)$ and $\mathrm{E}[\cdot]$ is the expectation operator. If $s(k)$ is Gaussian, $\kappa(s(k))$ is equal to zero. While the sign of the kurtosis determines the signal is sub-Gaussian or super-Gaussian, which corresponds the negative or positive. Sub-Gaussian signals are often arisen in wireless communication, radar and sonar [9,10], such as phase shift keying (PSK), quadrature amplitude modulation (QAM), and pulse amplitude modulation (PAM). In this case, the higher-order statistics can be utilized to improve the beamformer performance [11–13]. In [11], a minimum dispersion (MD) criterion is proposed to minimize the l_p-norm ($p \geq 1$) of the array output, and for sub-Gaussian signals, the performance can be improved significantly with $p > 2$. On the other hand, signals are often second-order (SO) noncircular and nonstationary in radio

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communication, such as binary phase-shift keying (BPSK), amplitude-shift keying (ASK), and unbalanced quaternary phase shift keying (UQPSK) signals. For this class of signals, the widely linear (WL) beamformer [14] is developed to exploit the noncircularity. Moreover, the WL-MVDR beamformer [15] is shown a better performance than the conventional beamformers. In [16], the optimal WL-MVDR beamformer, which leads to a further performance improvement, is proposed by exploiting the noncircularity of the desired signal [17,18].

Thus, for the class of sub-Gaussian signals encountered, which also exhibit noncircularity, such as BPSK, UQPSK and PAM signals, we propose two WL-MD based beamforming: widely linear minimum dispersion distortionless response (WL-MDDR) beamforming and widely linear quadratically constrained minimum dispersion (WL-QCMD) beamforming. The WL-MDDR beamformer is designed where we exactly known the steering vector (SV) and noncircularity coefficient of the desired signal. In the case of SV error and imprecise noncircularity coefficient, the WL-QCMD beamforming is developed to improve the robustness against the errors. The main contributions of our work contain: fully use the noncircularity and sub-Gaussian properties of signals, reserve the advantages of good performance at high signal-to-noise ratio (SNR) and be able to process more signals than the number of sensors.

2. Problem formulation

Considering an array of N sensors to receive narrowband signals, the array output $\mathbf{x}(k)\in\mathbb{C}^{N\times 1}$ is modeled as

$$\mathbf{x}(k)=s(k)\mathbf{a}+\mathbf{v}(k) \tag{1}$$

where $s(k)$, $\mathbf{a}$, $\mathbf{v}(k)$ are the desired signal, SV, and total interference-plus-noise vector, respectively. We assume that $\mathbf{x}(k)$ is noncircular and nonstationary, and the additive noise is Gaussian white process with zero-mean.

2.1. MDDR beamformer

For Gaussian signals and noise, since the MV criterion is statistically optimal, the MVDR beamformer is designed by minimizing the output variance while constraining the desired signal response to be unity, which can be formulated as

$$\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}_x\mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H\mathbf{a}=1 \tag{2}$$

where $\mathbf{R}_x=\left\langle E[\mathbf{x}(k)\mathbf{x}(k)^H]\right\rangle$ is the covariance matrix and time dependent. $\langle\cdot\rangle$ represents the time-averaging operation, with respect to the time index k, over the observation window [16], and $(\cdot)^H$ denotes the conjugate transpose.

For the non-Gaussian signals and noise, however, the higher-order or fractional lower order statistics may contain useful information, which can be utilized to improve the performance and leads to the MDDR beamformer [11]

$$\min_{\mathbf{w}} E\left[\left|\mathbf{w}^H\mathbf{x}(k)\right|^p\right] \text{s.t.} \quad \mathbf{w}^H\mathbf{a}=1 \tag{3}$$

where $E\left\{\left|\mathbf{w}^H\mathbf{x}(k)\right|^p\right\}$ is called as the dispersion and can be viewed as a generalization of variance. In the nonideal conditions, compared to the MVDR beamformer, the MDDR beamformer implicitly exploits the non-Gaussianity of the signals and results in a significant performance improvement with $p>2$ for the sub-Gaussian signals or $p<2$ for the super-Gaussian signals [11]. Moreover, a larger value of p gives better performance for the sub-Gaussian signals and the l_∞-norm can approach the upper bound of the SINR as the SNR increases. Note that when $p=2$, the MDDR beamformer reduces to the MVDR beamformer.

2.2. Optimal WL-MVDR beamformer

For the noncircular observation $\mathbf{x}(k)$, $\mathbf{C}_x=\left\langle E[\mathbf{x}(k)\mathbf{x}(k)^T]\right\rangle \neq \mathbf{0}$, where $(\cdot)^T$ denotes the transpose transpose, which can be used to improve the beamformer performance. Therefore, the extended observation vector is designed by:

$$\tilde{\mathbf{x}}(k)\triangleq[\mathbf{x}(k)^T,\mathbf{x}(k)^H]^T=s(k)\tilde{\mathbf{a}}_1+s(k)^*\tilde{\mathbf{a}}_2+\tilde{\mathbf{v}}(k) \tag{4}$$

where $\tilde{\mathbf{a}}_1=[\mathbf{a}^T,\mathbf{0}_N^T]^T$, $\tilde{\mathbf{a}}_2=[\mathbf{0}_N^T,\mathbf{a}^H]^T$, $\tilde{\mathbf{v}}(k)=[\mathbf{v}(k)^T,\mathbf{v}(k)^H]^T$. We assume that the desired signal $s(k)$ is noncircular. The noncircularity coefficient is defined as $\gamma_s=\left\langle \mathrm{E}[s(k)^2]\right\rangle/\pi_s$, where $\pi_s=\langle \mathrm{E}[|s(k)|^2]\rangle$ is the time-averaged power. $\gamma_s=|\gamma_s|e^{j\phi_s}$, $0\le|\gamma_s|\le 1$, where $|\gamma_s|$ and ϕ_s represent the noncircularity rate and phase, respectively. For $\gamma_s\neq 0$, $s(k)^*$ is correlated with $s(k)$ and contains both a desired signal and an interference component, which has the following orthogonal decomposition [16]:

$$s(k)^*=\gamma_s^* s(k)+[\pi_s(1-|\gamma_s|^2)]^{1/2}s'(k) \tag{5}$$

where $s'(k)$ with $\mathrm{E}[s(k)s'(k)^*]=0$ and $\mathrm{E}[|s'(k)|^2]=1$ denotes an orthogonal component of $s(k)$. Therefore, the extended observation vector is rewritten as

$$\begin{aligned}\tilde{\mathbf{x}}(k)&=s(k)\underbrace{(\tilde{\mathbf{a}}_1+\gamma_s^*\tilde{\mathbf{a}}_2)}_{\tilde{\mathbf{a}}_\gamma}+\underbrace{s'(k)[\pi_s(1-|\gamma_s|^2)]^{1/2}\tilde{\mathbf{a}}_2+\tilde{\mathbf{v}}(k)}_{\tilde{\mathbf{v}}_\gamma(t)}\\&=s(k)\tilde{\mathbf{a}}_\gamma+\tilde{\mathbf{v}}_\gamma(k)\end{aligned} \tag{6}$$

where $\tilde{\mathbf{a}}_\gamma$ and $\tilde{\mathbf{v}}_\gamma(k)$ are the equivalent extended steering vector (ESV) of the desired noncircular signal and the global noise vector for the extended observation vector $\tilde{\mathbf{x}}(k)$, respectively. The output of the WL beamformer is

$$y(k)=\tilde{\mathbf{w}}^H\tilde{\mathbf{x}}(k) \tag{7}$$

where $\tilde{\mathbf{w}}\in\mathbb{C}^{2N\times 1}$ is the WL weight vector. The optimal WL beamformer is designed based on the MV criterion as [16]

$$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}^H\mathbf{R}_{\tilde{v}_\gamma}\tilde{\mathbf{w}} \quad \text{s.t.} \quad \tilde{\mathbf{w}}^H\tilde{\mathbf{a}}_\gamma=1 \tag{8}$$

where $\mathbf{R}_{\tilde{v}_\gamma}=\left\langle \mathrm{E}[\tilde{\mathbf{v}}_\gamma(k)\tilde{\mathbf{v}}_\gamma(k)^H]\right\rangle$ and the optimal solution can be obtained as $\tilde{\mathbf{w}}_{\mathrm{MVDR}}=[\tilde{\mathbf{a}}_\gamma^H\mathbf{R}_{\tilde{v}_\gamma}^{-1}\tilde{\mathbf{a}}_\gamma]^{-1}\mathbf{R}_{\tilde{v}_\gamma}^{-1}\tilde{\mathbf{a}}_\gamma$.

However, the exact $\mathbf{R}_{\tilde{v}_\gamma}$ and $\mathbf{a}$ are unavailable in practical application, which are replaced by the extended sample covariance matrix $\hat{\mathbf{R}}_{\tilde{x}}=1/K\sum_{k=1}^{K}\tilde{\mathbf{x}}(k)\tilde{\mathbf{x}}(k)^H$, where K is the number of snapshots, and the presumed SV $\bar{\mathbf{a}}$. The output signal-to-interference-plus-noise ratio (SINR) of a