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A numerical approach to directly compute nonlinear normal modes of geometrically nonlinear finite element models

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ABSTRACT

The nonlinear normal modes of a dynamical system provide a modal framework in which the dynamics of a structure can be readily understood. Current numerical approaches use continuation to find a nonlinear normal mode branch that initiates at a low energy, linearized mode. The predictor-corrector based approach follows the periodic solutions as the response amplitude increases, forming the nonlinear normal mode. This method uses the Jacobian of the shooting function in a Newton–Raphson algorithm to find the initial conditions and integration period that result in a periodic response of the conservative equations of motion. Large scale finite element models require that the Jacobian be computed using finite differences since the closed form equations are not explicitly available. The Jacobian must be computed with respect to all of the states, making the algorithm prohibitively expensive for models with many degrees-of-freedom. In this paper, the initial conditions of each periodic solution are determined based on a subset of the linear modes of a geometrically nonlinear finite element model. The first approach, termed enforced modal displacement, sets the initial conditions as a linear combination of linear mode shapes. The second approach, here called the applied modal force method, applies a static load to the structure in a combination of applied forces that would excite a single linear mode, computes the static response to that load, and uses that to set the initial conditions. Both of these algorithms greatly reduce the number of variables that are iterated on during continuation. As a result, the cost of computing each solution along the nonlinear normal mode is only on the order of ten times the cost required to integrate the finite element model over one period of the response. The algorithm is initiated with only one linear mode and additional modes are added in a systematic way as they become important to the periodic solutions along the nonlinear mode branch. The approach is demonstrated on two geometrically nonlinear finite element models, showing a dramatic reduction in the computational cost required to obtain the nonlinear normal mode.

1. Introduction

In linear system theory, a normal mode of vibration provides a geometric tool that gives insight into the behavior of a dynamical system. Vibration modes provide a framework for model validation and updating, substructuring, system identification and structural health monitoring (to name a few). It is well known that linear theory is, however, an

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Nomenclature

$\mathbf{B}$	modal transformation matrix
ε	tolerance of shooting function
$\mathbf{f}_{NL}(\mathbf{x})$	nonlinear restoring force vector
$\mathbf{F}_{static}$	statically applied load for AMF method
$\mathbf{H}(T, \mathbf{q}_0)$	shooting function
$\mathbf{K}$	stiffness matrix
$\mathbf{M}$	mass matrix
$\mathbf{P}$	tangent prediction vector
$\mathbf{\Phi}$	linear mode shape matrix
$\mathbf{q}$	modal coordinate vector
s	step size controller
T	period
$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$	displacement, velocity and acceleration
$\mathbf{z}, \dot{\mathbf{z}}$	state space vectors
$()^T$	transpose operator

approximation to reality, since all real world engineering structures are inherently nonlinear. If a model contains structural nonlinearities, such as nonlinear material laws, large deformations, or contact, linear theory and linear normal modes no longer offer a valid mathematical model of the real physics. When nonlinearity cannot be circumvented, the concept of a nonlinear normal mode (NNM) provides a rigorous mathematical and geometric tool to describe the behavior of a system. This new definition of a mode can capture salient behavior not explained by linear theory, such as bifurcations, internal resonances, localization and frequency–energy dependence. This provides a natural extension to modal analysis for systems containing structural nonlinearities.

In extreme environments, many engineering structures must operate in their "nonlinear" range. For example, the skin panels of concept hypersonic aircraft vibrate nonlinearly due to large deformations in response to pressure fluctuations caused by air flow and engine noise during flight [1]. These panels also can vibrate about a buckled equilibrium state due to aerothermal loads [2–4], leading to highly nonlinear behavior even when the material is within its elastic limit. This makes their response and life difficult to predict since linear analysis methods are no longer valid. These panels require a careful design to assure that the structure can withstand the in-flight loads while not exceeding stringent weight requirements. When designing the subcomponents of an air vehicle, a finite element model is required to describe the detailed geometry of the structure (e.g. stiffeners, curved surfaces, etc…). The ability to compute the nonlinear normal modes of high fidelity models would help explain the physical behavior of these structures, and provide a useful design tool when nonlinearity is unavoidable. In this paper, two methods are proposed to compute the nonlinear normal modes of structures that are modeled within a commercial, geometrically nonlinear finite element package.

The earliest definition of a nonlinear normal mode was presented by Rosenberg in [5], in which the nonlinear mode of a conservative nonlinear system was essentially defined as a "vibration in unison". Each degree-of-freedom (DOF) of a nonlinear oscillator reaches its extreme points at the same instant in time, and simultaneously passes through zero. This definition was further developed by Vakakis [6] and Kerschen et al. [7] to account for modal interactions, termed internal resonances. They defined a nonlinear normal mode as a not necessarily synchronous periodic response of the undamped nonlinear system. A nonlinear mode describes the resonant frequency and deformation of the system as a function of the response amplitude (or conserved energy). Other definitions of nonlinear modes exist for conservative and nonconservative systems [8,9], but are not considered in this work. Although useful modal properties such as superposition and orthogonality are no longer valid with this new definition, NNMs still provide insight into the behavior of a nonlinear structure under a variety of operating environments. Nonlinear modes are related to the backbone of the structure′s nonlinear frequency response [7,10–12], and act as an attractor to the lightly damped free response [7,13,14]. They also have been used to create an amplitude dependent reduced order model to perform modal substructuring [15–17]. In those works, a subset of the NNMs of the subcomponents was used to predict the nonlinear modes of an assembly.

A variety of analytical techniques exist for computing nonlinear normal modes, such as the method of multiple scales [6,7,18,19] or the harmonic balance method [20]. These analytical methods may be restricting in practice since the formulation is mathematically intensive, and requires the equations of motion to be known in closed form. The advent of the finite element method allows engineers to model realistic structures within commercial software packages, where the equations of motion are generally not known in closed form. For this reason, a numerical approach to compute the nonlinear normal mode is considered in this work. Slater [21] developed a numerical algorithm that uses time integration, optimization and sequential continuation to track the periodic solutions of the undamped nonlinear system. The initial conditions and period of integration are optimized with a cost function that satisfies the periodicity condition. Peeters et al. [22] recently developed a similar algorithm based on pseudo-arclength continuation and the shooting technique, which is capable of capturing internal resonances (or fold bifurcations) along the NNM branch. The algorithm uses the equations of motion to analytically compute the Jacobian matrix using the approach in [23], and hence it has been successful with a structural model with hundreds of DOF [24]. Arquier et al. [25] developed a numerical technique to find the periodic solution of a conservative nonlinear system using a global solution technique, rather than shooting. The asymptotic numerical method is used as a continuation procedure to predict the next solution along the branch.

This paper presents a variant on the pseudo-arclength continuation algorithm by Peeters et al. [22] to compute the fundamental nonlinear normal modes of a finite element model within the native code, meaning that any software package can be used. The structural model is numerically integrated, using the available integration schemes, to a prescribed initial