Mechanical Systems and Signal Processing 46 (2014) 146–162

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Frequency-varying group delay estimation using frequency domain polynomial chirplet transform

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ARTICLE INFO

Article history:
Received 6 May 2012
Received in revised form
30 July 2013
Accepted 8 January 2014
Available online 1 February 2014

Keywords:
Group delay estimation
Polynomial phase signal
Polynomial chirplet transform
Time–frequency analysis
Lamb wave

ABSTRACT

In signal processing, group delay (GD) is used to measure the time delay of a signal passing through a system. It is an important parameter to be estimated for identifying, localizing, and tracking radiating sources. Moreover, it is conducive to acquire the characteristics and predict the response of the system. The GD is calculated by differentiating, with respect to frequency, the phase response versus frequency of the device under test, i.e., the slope of the phase response at any given frequency. In a linear phase system, the GD is constant. In practice, the GD of a signal passing through the system under test could vary with frequencies. Most GD estimation techniques are designed to estimate the constant GD instead of the frequency-varying GD. Time–frequency analysis is a potential tool for frequency-varying GD estimation. In this paper, a frequency domain polynomial chirplet transform (FPCT) is proposed. It first applies a GD-rotating operator and GD-shifting operator on the spectrum of the signal successively. Then, the inverse Fourier transform is performed on every windowed spectrum that is obtained by sliding the window throughout the shifted spectrum of the signal. An advantage of the FPCT is that it can obtain well-concentrated time–frequency representation so as to estimate nonlinear GD for wideband signals. The comparisons between the FPCT and other time–frequency analysis methods are provided to verify the performance of the FPCT in estimating the nonlinear GD. In addition, the study on experimental Lamb wave signal validates the effectiveness and potential of the FPCT.

1. Introduction

In signal processing, Group delay (GD) is an important parameter that is used to measure time delay of a signal passing through a system. It can be used to identify, localize, and track radiating sources, as well as acquire the characteristics and predict the response for the system under test. The GD is calculated by differentiating, with respect to frequency, the phase response versus frequency of the device under test, i.e., the slope of the phase response at any given frequency. In linear phase system, the GD is constant. In the past few decades, considerable advances in GD estimation have been developed. Most widely used methods are focus on the constant GD estimation, including generalized cross-correlation algorithm [1–3], the multichannel cross-correlation algorithm [4,5], and the blind channel identification technique based algorithms [6,7].

The GD of the signal passing through nonlinear phase system varies with frequencies. When a wideband signal passes through a system such as amplifier, a loudspeaker, or propagating through space or a medium, such as air, all frequency contents will be delayed. The wideband signals having frequency-varying GD appear in various fields, i.e., structure health

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0888-3270/$ - see front matter
http://dx.doi.org/10.1016/j.ymssp.2014.01.002

monitoring [8–12], seismology [13], physical optics [14,15], etc. Clearly, the traditional GD estimation techniques are incapable of estimating the frequency-varying GD of the wideband signal.

Time–frequency representation (TFR) is significantly effective to analyze non-stationary signals [16–18]. The TFR ridge is able to reveal the time–frequency signature of the signal, which can be characterized by the GD and instantaneous frequency (IF). Compared to GD, the IF is function of the time. The former would be the inverse function of the latter only when the variations in the time of IF are monotonic and the bandwidth-time (BT) product of signal is large [19]. Therefore, TFR is a promising method to estimate GD of the wideband response signal of the system. Various TFRs have been proposed to analyze non-stationary signals. Among them, wavelet transform (WT) and Wigner–Ville distribution (WVD), as typical non-parameterized TFR, attract more attentions. Although having many successful applications, WT cannot achieve high resolutions both in the time and frequency domains simultaneously due to uncertainty principle. The WVD is known for the best concentration, though it inevitably introduces plenty of cross-terms for signals with nonlinear time–frequency signature at low SNR. From post-processing point of view, reassignment technique [20,21] and the synchrosqueezing method [22] have been proposed to enhance the readability of TFR. The reassignment technique assigns the average of energy in a domain to the gravity center of these energy contributions. It sharpens blurry TFR at the cost of greater computational complexity. In the case of noise, the reassignment technique deforms the time–frequency profile as the computed gravity center unnecessarily represents the real time–frequency signature with less energy. As a special case of the reassignment method, the standard synchrosqueezing enhances the TFR along uni-axial, i.e., it summarizes the energy over a frequency range to be the energy of the center in the range at any given time. The standard synchrosqueezing fails to improve diffusion when the time–frequency signature of the signal varies quickly with time/frequency. To overcome such shortcomings, generalized synchrosqueezing transform [23] maps the signal with a curved instantaneous frequency to be a constant frequency prior to synchrosqueezing. Then, it recovers TFR for the signal from the wavelet transform on the mapped signal. Finally, the synchrosqueezing is performed on the removed TFR. The mapping function is determined by prior knowledge of the signal, which is impractical in real applications. Despite of this, the generalized synchrosqueezing transform results in excessive computational burden.

In contrast, parameterized TFR is more powerful to obtain a TFR that has fine resolution and is free from interference of cross terms [24–30]. However, since the kernel functions are designed to be a function of the time variable, most parameterized TFRs are only capable of characterizing the monotonic GD through estimating IF. For instance, chirplet transform (CT) is a typical parameterized TFR. With the properly determined chirp rate, the CT is able to have the energy of signal distributed closely along the real time–frequency signature of the linearly modulated signals. It is suitable to estimate linear GD of the signal with the linear kernel function. In order to adopt the CT to analyze guided wave signal with nonlinearly frequency-varying GD, several CT-based algorithms have been developed [9,26]. For instance, the group velocity of Lamb wave is dispersive with frequency, which can be calculated by multiplying the propagation distance with the GD of the signal. The CT-based algorithm essentially approximates the nonlinear GD with piecewise zero-order function of the time through searching the best chirplet basis or high-order chirplet locally. Similarly, in [13], a three-stage matching pursuit algorithm is adopted to analyze seismic wave signal. It selects the best matched wavelets from a redundant dictionary, which is still a piecewise zero-order approximation of the GD. These greedy search methods are neither effective nor smooth approximate of nonlinear GD. Besides, polynomial chirplet transform (PCT) [27], generalized warblet transform (GWT) [28] and spline chirplet transform [29] have been proposed recently. They are more effective and efficient to characterize the nonlinear IF of signal by using parameterized kernel function of the time. However, these parameterized time–frequency transforms are not able to characterize the frequency-varying GD that cannot be obtained from the inverse function of the IF.

Therefore, in this paper, a frequency domain polynomial chirplet transformation (FPCT) is developed to characterize the nonlinearly frequency-varying GD accurately and effectively. As the polynomial can approximate any bounded curve, the polynomial kernel of the FPCT is able to approximate arbitrary frequency-varying GD. In order to obtain well-concentrated TFR for a signal with frequency-varying GD, the FPCT first applies a GD-rotating operator on the spectrum of the signal, and second applies a GD-shifting operator. Then, it applies the short-frequency Fourier transform on the shifted signal spectrum. Moreover, we propose an iterative algorithm to estimate the parameters of the FPCT without little prior knowledge.

This paper is organized as follows: in Section 2, the FPCT is introduced in details. Section 3 contains the parameter estimation method for the FPCT and the performance comparison between the FPCT and other TFRs in GD estimation. In Section 4, the FPCT is applied on the experimental Lamb wave signal propagating in a plate. Section 5 draws the conclusion.

2. Frequency domain polynomial chirplet transform

2.1. Short-frequency Fourier transform and frequency domain chirplet transform

2.1.1. Short-frequency Fourier transform

For a signal $x(t)$, $x(t) \in L^2(R)$, the short-time Fourier transform is given by

$$\mathrm{STFT}(t_0, \omega_0) = \int_{-\infty}^{+\infty} x(t) w_\sigma^*(t-t_0) e^{-j\omega t} dt \tag{1}$$