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A hybrid expansion method for frequency response functions of non-proportionally damped systems

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ABSTRACT

This study is aimed at eliminating the influence of the higher-order modes on the frequency response functions (FRFs) of non-proportionally viscously damped systems. Based on the Neumann expansion theorem, two power-series expansions in terms of eigenpairs and system matrices are derived to obtain the FRF matrix. The relationships satisfied by eigensolutions and system matrices are established by combining the two power-series expansions. By using the relationships, an explicit expression on the contribution of the higher-order modes to FRF matrix can be obtained by expressing it as a sum of the lower-order modes and system matrices. A hybrid expansion method (HEM) is then presented by expressing FRFs as the explicit expression of the contribution of the higher-order modes and the modal superposition of the lower-order modes. The HEM maintains original-space without having to use the state-space equation of motion such that it is efficient in computational effort and storage capacity. Finally, a two-stage floating raft isolation system is used to illustrate the effectiveness of the derived results.

1. Introduction

The frequency response functions (FRFs) of mechanical and structural systems are of interest in dynamic problems subjected to harmonically varying loading that may be caused by reciprocating or rotating machine parts including motors, fans, compressors, and forging hammers [1,2]. FRFs are of fundamental importance and play a very important role in many areas such as model updating [3,4], structural damage detection [5,6], vibration and noise control [7,8], system identification [9–11], dynamic optimization [12,13] and many other applications.

Two kinds of methods, i.e., direct frequency response method (DFRM) and modal superposition method, are usually used to calculate the FRFs. The DFRM is based on the direct frequency results in an exact calculation by solving the inversion of the dynamic stiffness matrix directly. The modal superposition method calculates the FRFs by expressing them as the summation of the contributions of all the modes. The modal superposition method has been extensively used in many dynamic analyses, and also programmed in some commercial software, e.g., NASTRAN, ANSYS or ABAQUS. However, the modal superposition method requires that all the frequencies and mode shapes should be available. As we know, often it is difficult, or even unnecessary, to obtain all the eigenpairs of a large-scaled model, which means that the modal truncation scheme is generally used and the modal truncation error is therefore introduced. As a result, the quality of the calculated FRFs may be adversely affected.

The corrections to the modal truncation scheme have been investigated by several authors. Model acceleration methods [14–18] approximates the contribution of higher-order (unavailable) modes to dynamic response in terms of a pseudo-static

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term, which is a particular solution of the dynamic equation of motion (the particular solution can be considered as the exciting frequency equals zero). Since the mode acceleration method neglects the contribution of both velocity and acceleration terms to the dynamic response, it can be considered as a state approximation method. Dynamic correction methods [19–21], which were developed to improve the accuracy of mode acceleration method, include the contribution of higher-order modes by a sum of the particular solutions of both the equation of motion and the reduced differential equation of motion. Force derivative methods [22,23] reduce the modal truncation error by considering the higher-order derivatives of the forcing function, which means the forcing function should be described by analytical laws. The force derivative methods take advantage of the fact that successive integrations by parts of the convolution integral produce terms, which can be expressed as the combination form of system matrices, the forcing function and its derivatives. In addition, the correction methods for stochastic systems have been studied by Refs. [24–26] and can give a corrected response for both the deterministic and random forcing function. Bilello et al. [27,28] studied the corrected methods for continuous systems.

For the past two decades, high accurate modal superposition methods [29–33] were developed to the problem on the correction to the modal truncation scheme by combining the mode superposition of the lower (available) modes and a power-series expansion of dynamic response in terms of system matrices. These methods have been applied widely in the sensitivity of mode shapes [34–36] and the sensitivity of responses in the frequency domain [37–39]. Recently, Qu [40] presented an adaptive mode superposition and acceleration technique to solve the problem how many items of the convergent power-series expansion of dynamic response should be considered to satisfy the necessary accuracy. However, the high accurate modal superposition methods [36,39] approximate the FRFs using 2N-space (state-space) formulation, where N is the system dimension. Although these 2N-space correction methods are exact in nature, the 2N-space correction method usually needs heavy computational cost for real-life multiple degrees-of-freedom (DOF) systems since the size of system matrices of state-space equations is double. More recently, Li et al. [41] developed N-space correction methods to calculate the FRFs of nonviscously (viscoelastically) damped systems. The N-space correction methods attempt to approximate the influence of higher-order modes in terms of the lower-order modes and system matrices by using the first one or two terms of Neumann expansion of the contribution of higher-order modes. However, these procedures cannot be extended to further high-order terms since all of them will be affected by the nonviscous damping matrix which is frequency-dependent.

In this study, the correction problem of the modal truncation scheme of non-proportionally viscously damped systems is studied. The aim of this paper is to propose an N-space power-series expansion method to the problem on the correction of FRFs. Based on the Neumann expansion theorem, two power-series expansions in terms of eigenpairs and system matrices are derived to obtain the FRF matrix. By using the two power-series expansions, an explicit expression on the contribution of the higher-order modes can be expressed as a sum of the lower-order modes and system matrices. Then, a hybrid expansion method is presented by expressing the FRFs as the explicit expression of the contribution of the higher-order modes and the modal superposition of the lower-order modes.

The second section of this paper simply reviews the dynamic of non-proportionally viscously damped systems. The third section presents two power-series expansions to compute the FRF matrix. The fourth section gives some relationships between eigensolutions and system matrices, and presents an N-space power-series expansion method to the problem on the correction of FRF matrix and the displacement vectors. And the fifth section presents illustrate the engineering application, accuracy and efficiency of the presented method by a two-stage floating raft isolation system.

2. Dynamic of viscously damped systems

The dynamic equation of motion for a viscously damped system with N DOF in Laplace domain can be expressed as

$$(s^2\mathbf{M}+s\mathbf{C}+\mathbf{K})\mathbf{X}(s)=\mathbf{F}(s) \text{ or } \mathbf{D}(s)\mathbf{X}(s)=\mathbf{F}(s) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}\in\mathbb{R}^{N\times N}$are, respectively, the mass, damping and stiffness matrices (only consider symmetric system matrices in this study), $\mathbf{F}(s)$ is the forcing vector and $\mathbf{X}(s)$ is the displacement vector. The matrix $\mathbf{D}(s)=s^2\mathbf{M}+s\mathbf{C}+\mathbf{K}$ is so-called the dynamic stiffness matrix. In the context of structural dynamics, $s=i\omega$, where $i=\sqrt{-1}$ and ω denotes the exciting frequency in rad/s. The viscously damped system cannot be simultaneously decoupled by modal analysis unless it also possesses a full set of classical normal modes. The condition of viscously damped systems to possess classical normal modes (known as the proportionally damped system), originally introduced by Rayleigh [42] in 1877, is still extensively used. It shows that a viscous damping is proportionally damping if the damping matrix is a linear combination of inertia and stiffness matrices. This damping is routinely assumed in engineering applications. Later, Caughey and O'Kelly [43] and Adhikari [44] gave some more restrictive conditions which make damped systems possess normal modes as well. Generally speaking, proportional damping means that energy dissipation is almost uniformly distributed throughout the mechanical system [45]. However, there is no reason why these mathematical conditions must be satisfied. In practical, systems with two or more parts with significantly different levels of energy dissipation are encountered frequently in engineering designs. To this end, the non-proportionally damped system is considered in this study, i.e., the concern of this study is when these mathematical conditions are not met, the most general case in engineering applications.