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On the computation of the slow dynamics of nonlinear modes of mechanical systems

Malte Krack*, Lars Panning-von Scheidt, Jörg Wallaschek

Institute of Dynamics and Vibration Research, Leibniz Universität Hannover, 30167 Hannover, Germany

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ABSTRACT

A novel method for the numerical prediction of the slowly varying dynamics of nonlinear mechanical systems has been developed. The method is restricted to the regime of an isolated nonlinear mode and consists of a two-step procedure: In the first step, a multiharmonic analysis of the autonomous system is performed to directly compute the amplitude-dependent characteristics of the considered nonlinear mode. In the second step, these modal properties are used to construct a two-dimensional reduced order model (ROM) that facilitates the efficient computation of steady-state and unsteady dynamics provided that nonlinear modal interactions are absent.

The proposed methodology is applied to several nonlinear mechanical systems ranging form single degree-of-freedom to Finite Element models. Unsteady vibration phenomena such as approaching behavior towards an equilibrium point or limit cycles, and resonance passages are studied regarding the effect of various nonlinearities such as cubic springs, unilateral contact and friction. It is found that the proposed ROM facilitates very fast and accurate analysis of the slow dynamics of nonlinear systems. Moreover, the ROM concept offers a huge parameter space including additional linear damping, stiffness and near-resonant forcing.

1. Introduction

1.1. Motivation for reduced order modeling of nonlinear systems

In many structural dynamic systems, the effect of nonlinearity plays an important role. In fact, several engineering applications exploit nonlinear phenomena in order to improve the dynamic behavior of mechanical structures. Hence, there is a considerable need for efficient and versatile methods for the dynamic analysis of such systems.

For systems that comprise a large number of degrees of freedom (DOFs) and exhibit generic nonlinearities, as addressed in this study, the applicability of analytical methods is typically not possible and numerical methods have to be employed. The application of direct solution methods such as time-step integration often results in high computational costs. Therefore, extensive parametric studies, sensitivity and uncertainty analyses or design optimization soon become infeasible in conjunction with the full order model. Thus, there is a demand for Reduced Order Models (ROM) that are capable of significantly reducing the computational effort for the dynamic analysis and retaining the required accuracy of the predicted results.

* Corresponding author. Tel.: +49 511 762 4330.
E-mail address: krack@ids.uni-hannover.de (M. Krack).

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Nomenclature

a	modal amplitude
$\boldsymbol{f}$	nonlinear force vector
i	imaginary unit
$\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$	mass, damping, stiffness matrices
N_{h}	harmonic order of Fourier ansatz
t	time
$\boldsymbol{u}$	displacement vector
$\boldsymbol{u}_{\mathrm{p}}$	periodic form of displacement vector
$\boldsymbol{v}$	complex eigenvector
λ, ω_0, D	eigenvalue, eigenfrequency, damping ratio
Ω	angular frequency of oscillation
$\varphi, \varphi_{\mathrm{e}}$	fast phase, fast phase induced by excitation
$\boldsymbol{\Psi}_n$	N-th harmonic of eigenvector
Θ	slow phase
ϑ	absolute phase
$\dot{()}, \ddot{()}$	first- and second-order derivative with respect to time t
$\overline{()}$	complex conjugate
$()^{\mathrm{T}}$	transpose
$()^{\mathrm{H}}$	Hermitian transpose
$\langle \cdot, \cdot \rangle$	inner product
DOF	degree of freedom
FE	Finite Element
HBM	(high-order) Harmonic Balance Method
NMA	nonlinear modal analysis
ODE	ordinary differential equation
ROM	reduced order model

1.2. Existing approaches

Methods based on the invariant manifold approach [1–5] are widely used for the modal analysis of nonlinear mechanical systems as well as for the construction of efficient ROMs. The approach is based on the invariance property of certain periodic orbits of the dynamical systems, i.e. a nonlinear mode is defined as an invariant relationship (manifold) between several master coordinates and the remaining coordinates of the system. This manifold is typically governed by partial differential equations arising from the substitution of the manifold into the state form of the equations of motion. The invariant manifold approach was extended to account for the effect of harmonic excitation [3] and viscous damping [5]. It has been applied to various problems including piecewise linear systems [6], internally resonant nonlinear modes [4] and generic nonlinearly damped systems [7].

A straight-forward ROM strategy consists of constraining the system dynamics to the computed invariant manifold. This strategy is capable of drastically reducing the dimensionality of the problem to only a few master coordinates, while providing excellent accuracy for steady-state as well as unsteady dynamic predictions. One drawback is, however, the huge effort for the computation of the invariant manifolds. These computations can involve several thousands of nonlinearly coupled algebraic equations [3]. Moreover, the development of numerically robust algorithms for the treatment of generic, in particular non-conservative nonlinearities, seems to be an unresolved problem, see e.g. [7]. Furthermore, since the time-dependency is lost in the problem definition, the characteristic frequencies cannot directly be obtained from the computed manifold, but has to be identified from simulation results [6,7]. Finally, the parameter space of the ROM based on the invariant manifold approach is typically limited. For example, harmonic excitation and viscous damping are generally considered in the manifold computation step [3,5] so that even slight modification of these parameters would require the re-computation of the manifold.

Another category of methods for the determination of modal properties of nonlinear systems can be classified as nonlinear system identification (NSI) approaches [8–11]. Response data, obtained either by simulation or measurement, is gathered and modal properties are identified by fitting original response data to data from nonlinear modal synthesis. The weak point of this strategy is clearly its signal-dependent nature and the further effort required to obtain the response data. One of the main benefits of this method is that no model is required for the nonlinearities which enables broad applicability. Modal properties identified with NSI methods can be easily employed to feed the parameters of a nonlinear ROM [8,9].

Harmonic Balance approaches are widely used for the nonlinear modal analysis of conservative mechanical systems [12–14]. These methods are generally known to be well-suited for the analysis of strongly nonlinear systems with a large number of DOFs. Recently, Laxalde and Thouverez [15] extended the Harmonic Balance Method to the approximate modal analysis of dissipative systems by introducing a complex eigenfrequency in the Fourier ansatz. Krack et al. [16,17] applied this approach to various nonlinear mechanical systems and significantly improved the numerical performance of the analysis. This frequency-domain modal analysis concept allows for the direct calculation of iso-energy orbits on the invariant manifold as well as eigenfrequency and modal damping ratio of the nonlinear system. The modal properties have also been exploited in a ROM formulation for the prediction of steady-state vibrations [15–17]. Krack et al. [16,17] also investigated the limitations of the ROM and demonstrated that the validity of this approach is restricted to those regimes in which the energy is confined to a single nonlinear mode. This finding is completely in line with the work of Blanc et al. [18], who concluded that the absence of nonlinear modal interactions represents an intrinsic limitation to ROMs based on the invariant manifold concept.

1.3. Need for research regarding approaches for unsteady dynamics

For various engineering applications, it is important to assess the transient dynamics induced 'on the way to' the operating point, if it exists. Most of the above-mentioned ROM concepts are, however, designed to predict the steady-state