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Damping response analysis for a structure connected with a nonlinear complex spring and application for a finger protected by absorbers under impact forces

Takao Yamaguchi [a,*], Yusaku Fujii [b], Toru Fukushima [c], Norihisa Tomita [c], Akihiro Takita [b], Ken-ichi Nagai [a], Shinichi Maruyama [a]

[a] Department of Mechanical System Engineering, Faculty of Engineering, Gunma University, 1-5-1 Tenjin-cho, Kiryu, Gunma 376-8515, Japan

[b] Department of Electronic Engineering, Faculty of Engineering, Gunma University, 1-5-1 Tenjin-cho, Kiryu, Gunma 376-8515, Japan

[c] Graduate School of Department of Mechanical System Engineering, Faculty of Engineering, Gunma University, 1-5-1 Tenjin-cho, Kiryu, Gunma 376-8515, Japan

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ABSTRACT

This paper describes the dynamic response of a finger protected by a pair of viscoelastic absorbers under impact forces. The restoring forces of the finger and the absorbers are measured using the levitation mass method proposed by Fujii. In this study, we carry out numerical analysis of the dynamic response of a finger protected by absorbers under the same conditions as in the experiment. The absorbers and the finger are modeled by nonlinear concentrated springs using the power series of the elongations. Nonlinear complex spring constants are used to represent the changes in hysteresis as elongation progresses. This nonlinear spring is connected to a levitated block, which is modeled by three-dimensional finite elements. The experimental data are compared with the data calculated using our proposed finite element method.

1. Introduction

Isolation using concentrated springs has been utilized to protect lightweight structures (e.g. medical, electronic and precision apparatuses) from undesirable external vibrations and impacts.

However, sometimes, these lightweight structures are not very rigid. In such cases, the structures are regarded as elastic bodies.

Some concentrated springs used as isolating elements have nonlinear relations between the load and the displacement. Therefore, it is of great importance to clarify the dynamics of coupling between nonlinear springs and elastic bodies.

Many researchers have studied the nonlinear vibrations of concentrated masses with concentrated springs. For instance, Feeny studied this type of system using the proper orthogonal mode technique [6]. Vibration responses for large-scale problems, which involved multiple beams supported by nonlinear concentrated springs were investigated by Kondo, who also proposed a fast identification method for modeling nonlinear stability in large-scale problems [7]. Shaw used nonlinear modal analysis to investigate a simply supported beam attached to a nonlinear concentrated spring [8]. We previously

* Corresponding author. Tel.: +81 277 30 1577; fax: +81 277 30 1599.
E-mail address: yamagme3@gunma-u.ac.jp (T. Yamaguchi).

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Nomenclature

$\tilde{b}_i$ normal coordinates corresponding to $\{\varphi^{(i)}\}_0$
$\{\overline{d}\}$ vector containing nonlinear terms of the restoring force
$\{\hat{d}\}$ modified from $\{\overline{d}\}$
D_c displacement at the corner cube of the levitated block
D_g displacement at the center of gravity of the levitated block
$E=\overline{E}(1+j\eta_b)$ complex modulus of elasticity of the block
$\overline{E}$ real part of the E of the block
j imaginary unit
$[M_b],[K_b],\{f_b\},\{u_b\}$ mass matrix, complex stiffness matrix, nodal force vector and displacement vector of the block, respectively
$\{r\}=\{0,0,R\alpha z\}^T$ nodal force vector at α
$R\alpha z$ restoring force of the concentrated spring in the z direction at α
t time
$\{u\},[M],[K],\{f\}$ displacement vector, mass matrix, complex stiffness matrix and external force vector in the global system, respectively
$U_{\alpha z}$ displacement in the z direction at α
$\{U_s\}=\{U\alpha x, U\alpha y, U\alpha z\}^T$ nodal displacement vector at α
α nodal point where the nonlinear concentrated spring is connected with the elastic block
$\gamma_1=\overline{\gamma}_1(1+j\eta_{s1})$ linear complex spring constant of the nonlinear concentrated spring
$\overline{\gamma}_1$ real part of γ_1
η_{s1} material loss factor of the concentrated spring
$\gamma_2=\overline{\gamma}_2(1+j\eta_{s2}),\gamma_3=\overline{\gamma}_3(1+j\eta_{s3}),\gamma_4=\overline{\gamma}_4(1+j\eta_{s4}),\ldots$ nonlinear spring constant of the concentrated spring
$\overline{\gamma}_2,\overline{\gamma}_3,\overline{\gamma}_4,\ldots$ real parts of $\gamma_2,\gamma_3,\gamma_4,\ldots$, respectively
$\eta_{s2},\eta_{s3},\eta_{s4},\ldots$ nonlinear components of the material loss factor of the concentrated spring
$[\overline{\gamma}_1]$ complex stiffness matrix involving only the linear term of the restoring force
η_b material loss factor of the block
$\eta_{tot}^{(i)}$ i-th modal loss factor
$\omega_0^{(i)}$ i-th natural frequency
$\{\phi^{(i)}\}$ complex eigenvector
$\{\phi^{(i)}\}_0$ approximated linear natural modes
$\phi_{\alpha iy}$ y-component of the eigenmode $\{\phi^{(i)}\}_0$ at α

proposed a fast numerical method for computing nonlinear vibrations in an elastic block or a viscoelastic block with a nonlinear concentrated spring when one end of the nonlinear spring is connected to the ground [2,3]. Furthermore, we extended the method to describe the dynamic phenomena of viscoelastic blocks connected with a nonlinear concentrated spring having a restoring force expressed as a function of the relative displacement between the two ends of the spring [4]. In our numerical analysis, the discrete equations in physical coordinates are transformed into nonlinear ordinary coupled equations using normal coordinate corresponding to linear natural modes. In this process, modal damping is also transformed. The transformed equations are integrated numerically with an extremely small degree-of freedom.

Absorbers often have nonlinear hysteresis among their dynamic properties. For instance, hysteresis increases as the elongation of the absorbers increases [5]. However, few researchers have investigated coupled vibration between elastic structures and absorbers with nonlinear hysteresis.

Here, we use the finite element method (FEM) to extend our proposed method of vibration analysis to elastic structures with nonlinear concentrated springs with nonlinear hysteresis. The restoring force of the spring is expressed as the power series of its displacement. The restoring force also includes nonlinear hysteresis damping. Therefore, complex stiffness is introduced for not only the linear but also the nonlinear components of the restoring force. The finite element for the spring is expressed and is connected to elastic structures modeled by linear solid finite elements. The discrete equations in the physical coordinates are transformed into nonlinear ordinary coupled equations using normal coordinates corresponding to linear natural modes. In this process, modal damping is also transformed. The transformed equations are integrated numerically with an extremely small degree-of freedom. This numerical method is used to study a finger protected by absorbers under impact forces as follows.

To protect human body parts, such as arms, fingers, and feet from getting caught in the door of mechanical apparatuses, transporters or buildings, viscoelastic rubbers are sometimes inserted between doors and their frames. These viscoelastic rubbers have rolls of shock absorbers to decrease the impact. The rubbers come in many shapes to maximize the performance of the shock absorbers. For example, some thin viscoelastic rubbers with hollow cross sections use buckling phenomena to decrease impact. Therefore, viscoelastic absorbers have a nonlinear restoring force under a relatively large load. These absorbers often have nonlinear hysteresis among their dynamic behaviors. Therefore, it is important to clarify the nonlinear dynamic properties of viscoelastic shock absorbers with elastic structures under an impact load.

Here, we use FEM with the nonlinear complex springs to apply our proposed method to this problem. In the numerical analysis, the absorbers are modeled as the viscoelastic nonlinear springs with nonlinear hysteresis. The restoring force of the spring is expressed as the power series of its elongation. The restoring force also includes nonlinear hysteresis damping. By colliding a block and a pair of viscoelastic shock absorbers with and without a finger, we can use the levitation mass method proposed by Fujii [13], to measure the velocity of the block. In this previous paper [13, 1], we treated numerical simulation of dynamic responses for a viscoelastic absorber without a finger. This previous paper did not include calculated