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Partial eigenvalue assignment for high order system by multi-input control

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ABSTRACT

An algorithm of partial eigenvalue assignment problem for high order systems is given that the spectrums are partially reassigned to predetermined locations and the remaining spectrums keep unchanged. The algorithm requires the knowledge of only a small number of eigenvalues and their corresponding eigenvectors. Numerical examples are done to illustrate the effect of the approach.

1. Introduction

Consider the high order dynamical system

$$P_K(D)v(t) = f(t), \tag{1.1}$$

where $P_K(D) = \sum_{k=0}^{K} M_k D^k$, $D^k = d^k/dt^k$ is a differential operator, M_k is real constant $n \times n$ matrices, $k = 0, 1, \ldots, K$, and M_K is nonsingular. Using separation of variables of the form $v(t) = xe^{\lambda t}$ in Eq. (1.1), where x is a constant vector, we obtain the high order eigenvalue problem

$$P_K(\lambda)x = 0, \tag{1.2}$$

where

$$P_K(\lambda) = \sum_{k=0}^{K} M_k \lambda^k. \tag{1.3}$$

The characteristic roots λ of the polynomial Eq. (1.3), namely det $[P_K(\lambda)] = 0$, are known as eigenvalues. The corresponding vectors $z \neq 0$ and $x \neq 0$ are corresponding left and right eigenvectors which respectively satisfy

$$z^H P_K(\lambda) = 0, \quad P_K(\lambda)x = 0, \tag{1.4}$$

where H is the conjugate transpose.

It is well known that if the Kn eigenvalues $\{\lambda_i\}_{i=1}^{Kn}$ of Eq. (1.3) satisfy $\mathrm{Re}(\lambda_i) \leq 0$ for all $i = 1, 2, \ldots, Kn$, then the response of Eq. (1.1) is bounded for arbitrary initial conditions. The response of the system to initial conditions is required in some applications to diminish rapidly. This objective can be achieved by relocating eigenvalues of the system in the

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complex plane. Assume that we wish to alter the location of the eigenvalues by applying the control force $f(t)=Bu(t)$, where B is an $n \times m(m \le n)$ real matrix and $u(t)$ is a time-dependent $m \times 1$ real vector. Matrix B is known as the control matrix, and without loss of generality, we assume that B has full column rank, that is, rank $(B)=m$. The special choice

$$u(t)=\sum_{j=0}^{K-1} F_{K-j}^T D^j v(t), \tag{1.5}$$

where $\{F_i\}_{i=1}^K \in R^{n\times m}$ is called a state feedback control, and leads to the closed-loop eigenvalue problem

$$P_c(\eta)=P_K(\eta)-Bh^T(\eta), \quad h(\eta)=\sum_{j=0}^{K-1} \eta^j F_{K-j}. \tag{1.6}$$

Many practical situations where such problem arises and needs to be solved are, for example, vibration analysis of structural mechanical and acoustic system, electrical circuit simulation, fluid mechanics, finite element model updating in aerospace and automobile industries. The partial eigenvalue assignment problem is extensively considered by researchers. Especially, the problem of partial eigenvalue assignment for high order system has caused wide attention in engineering. It has been widely applied in many fields such as civil seismic, vibration suppression of mechanical structure and shock absorption of rail vehicles. Mathematically, the problem is then to choose the matrices $\{F_i\}_{i=1}^K \in R^{n\times m}$ such that the eigenvalues of Eq. (1.6) can be altered as required.

In most practical situations, however, only a few eigenvalues of Eq. (1.3) are undesirable, so it makes more sense to alter only those undesirable eigenvalues, while keeping the rest of the spectrum invariant. This leads to the following problem, known as the partial eigenvalue assignment problem for high order system.

Problem 1. Given $n \times n$ real matrices $\{M_K, M_{K-1}, \ldots, M_0\}$ with M_K nonsingular, the $n \times m$ real control matrix B, the self-conjugate subset $\{\lambda_i\}_{i=1}^p (p<n)$ of the open-loop spectrum $\{\lambda_i\}_{i=1}^{Kn}$ and the corresponding eigenvector set $\{x_i\}_{i=1}^p$ and given a self-conjugate set $\{\mu_i\}_{i=1}^p$, find $n \times m$ real feedback matrices $\{F_i\}_{i=1}^K$ such that the spectrum of the closed-loop pencil $P_c(\lambda)$ in Eq. (1.6) is $\{\eta_i\}_{i=1}^p \cup \{\lambda_i\}_{i=p+1}^{Kn}$.

It is known that when $m=1$, the solution to Problem 1 may be solved uniquely by single-input state feedback control. Mohamed A. Ramadan [1] has solved the problem of assigning the desired eigenvalues without altering the rest of the spectrum. However, we must recognize that the single-input method usually needs large control force. A typical application arising from second-order control system has been researched by lots of scholars, see [2–8]. For solving quadratic pole assignment by multi-input control Y.M. Ram and S. Elhay have given a multi-step method in [9]. Their method considered there can be used in quadratic symmetric control system. Though on the practical side, the extraction of left eigenvectors by modal test is a sensitive process prone to measurements errors, we develop the multi-input multi-step solution for solving partial pole placement of a linear differential equations without the restriction on order. The approach in this paper works directly in high order system without turning to first order one. It will be explained why a significant reduction in the control forces can be achieved and the numerical examples illustrate the feedback matrices to be computed in such a way that their norms are smaller than the result in [9]. For an application we give a practical multi-input mechanical system described by the dynamic equation

$$\ddot{x}+\begin{pmatrix} b_1c_1 & b_2c_2 \\ b_1c_2 & b_2c_1 \end{pmatrix}\dot{x}+\begin{pmatrix} k_1c_1 & k_2c_2 \\ k_1c_2 & k_2c_1 \end{pmatrix}x=\begin{pmatrix} c_1 & c_2 \\ c_2 & c_1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix},$$

where the state vector $x(t)=[x_1(t), x_2(t)]^T$, $c_1=m^{-1}+L^2/I$ and $c_2=m^{-1}-L^2/I$ in which m and I represent the mass and inertia of the mass, k_1 and k_2 are the spring constants, b_1 and b_2 are the damper constants, x_1 and x_2 are the mass displacement from both sides, $0.5(x_1+x_2)$ is the vertical displacement of the center of mass, $0.5(x_1-x_2)/L$ is the inclination small angle of the mass with the horizontal, $2L$ is the distance between two supporting points, and u_1 and u_2 are the control inputs. The configuration of this mechanical system and its parameters has been shown by Fig. 1. in [11].

The paper is organized as follows. In Section 2 we develop the method of Y.M. Ram [10] to solve partial eigenvalue assignment in high order system. We then consider the partial assignment of eigenvalues by multi-input control in Section 3. Numerical examples demonstrating significant reductions in the magnitude of the control forces by using multi-input control are presented in Section 4. Conclusions are finally drawn in Section 5.

2. Partial eigenvalue assignment for high order system by single-input control

For single-input control application, the system is modified by applying a controlling force $f(t)=bu(t)$ as

$$P_K(D)v(t)=bu(t), \tag{2.1}$$

where $b \in R^n$ is a constant vector, and the location of control and associated control force $u(t)$ is defined as

$$u(t)=\sum_{j=0}^{K-1} f_{K-j}^T D^j v(t). \tag{2.2}$$