Mechanical Systems and Signal Processing 42 (2014) 247–257

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Partial eigenvalue assignment and its stability in a time delayed system

Kumar V. Singh [a,*], Rajeeb Dey [b], Biswa N. Datta [c]

[a] Mechanical and Manufacturing Engineering Department, Miami University, Oxford, OH 45056, USA
[b] Electronics and Communication Engineering Department, Sikkim Manipal Institute of Technology, Majitar, Sikkim 737136, India
[c] Department of Mathematical Sciences, Northern Illinois University, De Kalb, IL 60115, USA

ARTICLE INFO

Article history:
Received 1 April 2013
Received in revised form
26 August 2013
Accepted 29 August 2013
Available online 20 September 2013

Keywords:
Active vibration control
Partial pole placement
Time delay
Stability

ABSTRACT

Active vibration control strategy is an effective way to control dangerous vibrations in a structure, caused by resonance and to manipulate the dynamics of vibrational response. Implementation of this strategy requires real-time computations of two feedback control matrices such that a small amount of eigenvalues of the associated quadratic matrix pencil are replaced by suitably chosen ones while the remaining large number of eigenvalues and eigenvectors remain unchanged ensuring the no spill-over. This mathematical problem is referred to as the Quadratic Partial Eigenvalue Assignment problem. The greatest challenge there is to solve the problems using the knowledge of only a small number of eigenvalues and eigenvectors that are computable using state-of-the-art techniques. This paper generalizes the earlier work on partial assignment to constant time-delay systems. Furthermore, a posterior stability analysis is carried out to identify the ranges of the time-delay that maintains the closed-loop assignment while keeping the stability of the infinite number of eigenvalues for the time-delayed systems. The practical features of the proposed methods are that it is implemented in the second-order setting itself using only those small number of eigenvalues and the eigenvectors that are to be assigned and the no spill-over is established by means of mathematical results. The results of our numerical experiments support the validity of our proposed methods.

1. Introduction

Large complex vibrating structures such as automobiles, rotors, air/space crafts, buildings, bridges, etc. are generally modeled or approximated by using finite element or finite difference techniques to a system of second order matrix differential equations of the following form:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{0}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}\in\Re^{n\times n}$ are known as the mass, damping and stiffness matrices respectively. Very often these matrices are symmetric, $\mathbf{M}=\mathbf{M}^T$, $\mathbf{C}=\mathbf{C}^T$, $\mathbf{K}=\mathbf{K}^T$ and furthermore $\mathbf{M}>0$ and $K\geq 0$. The use of the separation of variables of the form $\mathbf{q}(t)=\mathbf{x}e^{\lambda t}$, in (1) leads to the quadratic eigenvalue problem:

$$\mathbf{P}(\lambda_k)\mathbf{x}_k=0,\ \text{for } k=1,2,\cdots,2n, \tag{2}$$

* Correspondence to: Mechanical and Manufacturing Engineering Department, 650 E. High Street, EGB 056L, Miami University, Oxford, OH 45056, USA. Tel.: +1 513 529 0724 (O); fax: +1 513 529 0717.
E-mail address: singhkv@MiamiOH.edu (K.V. Singh).

0888-3270/$ - see front matter
http://dx.doi.org/10.1016/j.ymssp.2013.08.028

where

$$\mathbf{P}(\lambda) = \lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K}, \tag{3}$$

is the open loop quadratic pencil, $\{\lambda\}_{k=1}^{2n}$ are the eigenvalues or natural frequencies and $\mathbf{x}_k \neq 0$ for $k = 1, 2, \cdots, 2n$ are the corresponding eigenvectors or mode shapes of the vibratory system. The above second order system or the associated quadratic pencil $\mathbf{P}(\lambda)$ is often denoted by the triplet ($\mathbf{M}$,$\mathbf{C}$,$\mathbf{K}$).

The dynamics of the vibrating structure are determined from the eigenvalues and eigenvectors of the quadratic pencil (3). From the knowledge of these quantities, the system responses as well as stability can be determined. By assigning eigenvalues further to the left-hand side of the complex plane: (i) the transient motion of the system can be manipulated, (ii) the damping of the system can be enhanced, and (iii) the natural frequencies of the system can be kept away from resonance with the applied time varying harmonic loads. Such an assignment can be achieved either by passive control or by means of active control [1–4].

Passive devices are economic and widely used in practice; however there are some limitations, for example, they are single frequency devices and they can control only a limited range of vibration amplitude/frequencies. They are not suitable for a dynamically changing system. On the other hand, the active vibration control (AVC) techniques [5–9] are capable of supplying the desirable control forces in a dynamic environment (real time). For AVC application, the system (1) is modified by applying a suitable control force $\mathbf{Bu}(t)$. The closed loop controlled system then becomes,

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{B}\mathbf{u}(t), \tag{4}$$

where $\mathbf{B} \in \mathbb{R}^{n\times m}$ is the control distribution matrix, and $\mathbf{u}(t)$ is the associated control force vector given by,

$$\mathbf{u}(t) = \mathbf{F}^T\dot{\mathbf{q}}(t) + \mathbf{G}^T\mathbf{q}(t). \tag{5}$$

The implementation of AVC requires the constructions of the two feedback control matrices $\mathbf{F} \in \mathbb{R}^{n\times m}$ and $\mathbf{G} \in \mathbb{R}^{n\times m}$ such that the closed loop system

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{C} - \mathbf{B}\mathbf{F}^T)\dot{\mathbf{q}}(t) + (\mathbf{K} - \mathbf{B}\mathbf{G}^T)\mathbf{q}(t) = \mathbf{0}, \tag{6}$$

has the desired system responses. The dynamics of the closed loop systems are governed by the eigenvalues and eigenvectors of the quadratic matrix pencil,

$$\mathbf{P}_c(\tilde{\mu}) = \tilde{\mu}^2\mathbf{M} + \tilde{\mu}(\mathbf{C} - \mathbf{B}\mathbf{F}^T) + (\mathbf{K} - \mathbf{B}\mathbf{G}^T), \tag{7}$$

where, $\{\tilde{\mu}_k\}_{k=1}^{2n}$ are the closed loop eigenvalues of the controlled system associated with the pencil (7).

The instability or the dangerous vibration in a structure can be controlled by replacing few of the unwanted eigenvalues $\{\lambda_i\}_{i=1}^{p}$ to suitably chosen location, while keeping rest of the eigenvalues and the corresponding eigenvectors unchanged. This can be achieved by choosing the F and G matrices appropriately. Mathematically it is a quadratic partial eigenvalue assignment problem (QPEVAP) which can be stated as follows:

Given the triplet of matrices ($\mathbf{M}$,$\mathbf{C}$,$\mathbf{K}$) of the open loop system (1), the control input matrix B, a set of eigenvalues $\{\lambda_i\}_{i=1}^{p}$, $p \ll 2n$ that need to be reassigned, the set of suitably chosen p numbers $\{\tilde{\mu}_i\}_{i=1}^{p}$, find the feedback matrices F and G in such a way that the closed-loop spectrum becomes $(\tilde{\mu}_1, \cdots, \tilde{\mu}_p, \lambda_{p+1}, \cdots, \lambda_{2n})$, while the eigenvectors corresponding to $\lambda_{p+1}, \cdots, \lambda_{2n}$ remain unchanged.

Since the introduction of QPEAVP in the literature by Datta et al. [10] and its elegant mathematical solution to the single input case (B is a vector), much research has been done in recent years. These works have extended the single input solutions to the multi-input case [11,12] and also to the solution of eigenstructure assignment problem [13]. Furthermore, the aspects of robust feedback stabilization and the norm minimization have been considered and solved by the sophisticated techniques of numerical linear algebra and numerical optimization [14,15].

Unfortunately, the above work did not take into account of the practical aspects of the problem; namely the time delay in control. Though there exist a substantial amount of work on the stability of a time delayed system, the work on partial eigenvalue assignment in time delay is just emerging. Ram et al. [16] have solved the partial pole assignment with time delay for the single input case. Their solution requires both the knowledge of receptances as well as system matrices. Recently this hybrid approach has been extended by Bai et.al.[17] to the multi-input case. In either of these works the stability of the closed loop systems have not been investigated. The main difficulty with the time delayed problems is that the characteristic equation of the closed loop system is a transcendental function and hence the dynamic behavior of the closed loop system is characterized by infinitely many eigenvalues. Reassignment of few eigenvalues $\{\lambda_i\}_{i=1}^{p}$ to suitably chosen location $\{\mu_i\}_{i=1}^{p}$ and guaranteeing the no-spillover in the remaining ($2n$-p) eigenvalues of the open loop system cannot ensure the stability of the overall closed loop system. Numerous efforts have been made in the past to study the effect of time delay on the stability of the system as well as the estimation of the associated closed loop eigenvalues [18–23]. However, most of these stability analyses have not been done in the framework of partial eigenvalue assignment.

The main contributions of this paper are as follows:

- A new algorithm for the time-delayed QPEVAP is proposed. This algorithm generalizes our earlier work on the single input case [24]. The algorithm directly works in a given second order system without a-priori transformation to a standard state space system. It is implemented with the help of only a small number of eigenvalues and eigenvectors that