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Decoupling analysis on nonlinear system based on the modified generalized frequency response functions

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ABSTRACT

This paper presents a nonlinear decoupling approach based on the Modified Generalized Frequency Response Functions (MGFRFs) and the nonlinear feature of phase invariance, for the pure nonlinearity-input nonlinear system. The MGFRFs are defined by combining the '*homotopy*' GFRFs and phase information of the system input. The nonlinear feature of phase invariance is extracted based on MGFRFs. The decoupling approach is proposed based on MGFRFs and extended from the pure tone excitation to the multi-tone excitations by considering phase invariance. Numerical simulation and experimental investigation were carried out, whose results have shown that nonlinear feature of phase invariance is correct and reasonable and the proposed decoupling approach is valid and feasible. The proposed decoupling approach can be employed to identify the excitation sources and to estimate nonlinear system parameters for the pure nonlinearity-input nonlinear vibration system.

1. Introduction

Identification of vibration and noise source is prior to transfer path analysis. Both methods are the basis of improving vibration and noise reduction technology for the practical system, e.g. ships, submarines and aircrafts. There are many identification methods to cope with the sources when the sources are uncorrelated. Bendat et al. [1] introduced both coherence analysis and partial coherence analysis into the white noises MIMO system. Zhang et al. [2] evaluated underwater noise source contribution of ships by using partial coherence analysis on cross-spectral matrix data. Belouchrani et al. [3] presented joint diagonalization of covariance matrix of two order statistics. Blind source separation (BBS) is one of the most effective methods when the sources are uncorrelated and statistically independent. Hyvarinen et al. [4] proposed independent component analysis (ICA) based on high order statistics and Shannon entropy, and consequently conditionally ICA, kernel ICA and topological ICA came forth on BBS [4,5].

When nonlinearity is introduced into linear system, nonlinear behavior is the rule in the dynamic behavior of the physical system [6]. Mathematical modeling is one primary approach to understand nonlinear behavior, which obtained the representing model of the complete system. However, system identification plays more crucial role because it helps the structural dynamicist to reconcile numerical predictions with experimental investigations, to extract the information about the structural behavior from experiment data [7], and to estimate the parameters of experimental system. The methods of nonlinear system identification in structural dynamics are classified into seven categories, namely

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Nomenclature

a	linear output coefficient
b, c	(nonlinear) input coefficient
d	differential operator
e	natural logarithmic base
e	relative error
f	frequency
g, h	Volterra kernel
j	imaginary unit
i, j, k, l	positive integer
m	white Gauss noise; positive integer
n	white Gauss noise; positive integer
p	differential order
t	time
u	input of system
y	output of system
F	primary function of integral transform
$H, \overline{H}$	(generalized) frequency response function
$\hat{H}$	the modified generalized frequency response function
$\hat{\boldsymbol{H}}$	the modified generalized frequency response function matrix
M	the order of harmonic frequency; positive integer
N	the truncation order of nonlinear system; positive integer
$\mathbb{N}$	the set of natural numbers
U	frequency-domain input
$\hat{U}$	the modified frequency-domain input
$\hat{\boldsymbol{U}}$	the modified frequency-domain input vector
Y	frequency-domain output
$\boldsymbol{Y}$	frequency-domain output vector
α	amplitude
β	angular variable
π	the ratio of the circumference
τ	time
φ	phase
ω	angular frequency
Ψ	complex function
Ω	normalized frequency
int	integrate function

by-passing nonlinearity: linearization, time and frequency-domain methods, modal methods, time–frequency analysis, black-box modeling and structural model updating [7]. And some types of the models are employed in the process of nonlinear system identification, e.g. Volterra series [8], Fourier series [9], Hammerstein model [10], Wiener model [11], Hammerstein–Wiener model [12], and the orthogonal polynomial model [13].

In the identification process, once nonlinear behavior has been characterized, the parameters of the selected model are estimated using some linear or optimization algorithms depending upon the method considered [7]. Volterra series is a direct generalization of the linear convolution integral and has been widely applied in the analysis and design of nonlinear systems, both in time [14] and frequency [15] domain. Although Volterra series is a technique suitable for describing higher order Frequency Response Functions (FRFs) [14], it requires a large number of multidimensional coefficients to accurately model a nonlinear system, which makes it computationally intensive and complex [16]. Representation and identification of an all-purpose Volterra series is almost impossible [17]. Hence, in many practical situations, a truncated Volterra series is utilized to model a system characteristic with nonlinearities. The truncation order and its effect in Volterra series have been studied, e.g. Jing [18] established the analytical relationships among truncation order, excitation magnitudes, and estimation error for the output spectra of nonlinear systems. Moreover, Volterra series suffers from the problem of limited convergence [19,20] while the truncated model may lead to the errors [18,19]. The convergence aspects of Volterra series have been investigated, e.g. Barrett [21] proposed a time domain criterion to prove that Volterra series converges within a given region for a class of nonlinear systems with cubic stiffness nonlinearity; Li and Billings [22] extended this time-domain criterion to frequency-domain to accommodate the analysis of nonlinear oscillators; Sandberg [23] showed that a truncated model provides a uniform approximation to the infinite Volterra series on a ball of bounded input for a large class of systems; Chatterjee and Vyas [19] founded that the convergence limitations of Volterra series of Duffing oscillator under harmonic excitation are a function of the non-dimensional non-linear parameter and also dependent on the number of terms considered in the response series.

However, at present, there is no existing general method to calculate Volterra kernels for nonlinear systems, although they can be calculated for systems whose order is known and finite [16]. In time domain, the estimation of Volterra kernels has been studied based on block-oriented nonlinear structures [24–26], Wiener series or the other type of orthogonal functions [14,27], adaptation estimate methods [16,28] and the factorization method [29]. Kibangou and Favier proposed the estimation of the diagonal coefficients of Volterra kernels associated with Wiener–Hammerstein models [24] and parallel-cascade Wiener models [25], and developed a tensor analysis-based approach [26] for optimizing the parameters of block-oriented nonlinear structures. Dewson et al. [27] demonstrated that the orthogonal representation of Volterra kernels is directly described by the time series moments. Silva et al. [14] suggested an approach based on the identification of the 1st- and 2nd-order Volterra kernels in an orthogonal basis. Singh and Chatterjee [16] carried out estimation of truncated 2nd-order Volterra kernels by employing several adaptation algorithms. Brenner and Xu [29] developed an efficient factorization method that reduces a higher order Volterra kernel to a product of Volterra kernels of order one.

The frequency-domain version of Volterra kernels, called Generalized FRFs (GFRFs, linear and higher order FRFs), which can be obtained by taking the multiple Fourier transform of Volterra kernels, has also been extensively studied [30]. Worden et al. [31] extended single-input Volterra series to multi-input Volterra series through definition of direct and cross-kernels.