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Modal parameter extraction based on Hilbert transform and complex independent component analysis with reference

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A B S T R A C T

A novel method for extracting modal parameters, including mode shapes, natural frequencies and damping ratios, is developed in this study. By applying Hilbert Transform (HT) and complex Independent Component Analysis (ICA) with reference for Blind Source Separation (BSS), the modal parameters can be directly extracted while other articles focus on the mode shapes only. Moreover, a revised version of fixed-point complex ICA with reference for BSS is derived for a better separating performance with higher separation accuracy by comparison. The simulation and experiment results are both presented.

1. Introduction

Widely used in health monitoring, damage detection [1–3] and so on, the modal analysis can provide an insight into structure dynamical properties. Hence, lots of literatures are on the algorithms for modal parameter extraction, such as ARMA/TVARMA [4–7], subspace-based methods [8–11] and the popular Hilbert–Huang Transform [12–14]. No matter which method ones employ, the modal parameters are submerged in the vibration signals when dealing with experimental modal analysis. Fortunately the procedure of extracting modal parameters coincides with the original idea of BSS, which is developed to recover a set of underlying sources from observations without knowing the mixing procedure and sources. Research work has been done to investigate the application of BSS on the topic of modal parameter extraction [15–18]. In all of the four reference literatures, the mode shapes are mainly discussed while the natural frequencies are extracted by FFT-based peak-picking method. Moreover the analysis results of natural frequencies are used to determine the order of the mode shapes in turn because the sources are not separated in orders. If a priori knowledge is introduced, complex ICA for BSS would be expected to get a better separating performance and then a better modal parameter extraction result would be obtained. Hence, a modified version of complex fixed-point ICA with reference for BSS is applied. By adopting HT to convert a signal into the analysis form and complex ICA with reference to separate the observation signals into sources, the modal parameters including the mode shapes, the natural frequencies and the damping ratios can be extracted simultaneously, and the orders of modal parameters are determined in advance with the assistance of reference signals.

The correspondence is organized as follows. Section 2 briefly reviews the HT. Complex ICA with reference for BSS is derived in Section 3. The modal parameter extraction method based on HT and complex ICA with reference is presented in

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Section 4 in detail. Sections 5 and 6 focus on the simulation and experiment analysis, respectively. Conclusions and further investigations are drawn in Section 7.

2. Overviews on Hilbert transform

The HT is one of the integral transforms, which is first employed to solve a special case of the integral equations in the field of mathematical physics. Now it is widely used in signal processing. The HT of a signal $x(t)$ is defined by an integral transform [19]

$$H[x(t)] = \tilde{x}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{x(\tau)}{t-\tau}\,\mathrm{d}\tau \tag{1}$$

The mathematical definition in Eq. (1) really does not give much insight into the understanding and application of the HT. However, the physical meaning of the HT helps us to get a much deeper access to the transformation. Physically, the HT is an equivalent to a special kind of linear filters, with which all the amplitudes of the spectral components are remained unchanged, but their phases are shifted by $-\pi/2$ [12], e.g. $\cos(\omega t)\overset{H}{\rightarrow}\sin(\omega t)$. It can be viewed as that the HT of a signal $x(t)$ produces a signal $\tilde{x}(t)$ that is orthogonal to $x(t)$ [20]. Eq. (1) qualifies HT lots of good properties. And herein the primary properties are shown as follows:

$$\cos(\omega t)\overset{H}{\rightarrow}\sin(\omega t)\ \text{or}\ \sin(\omega t)\overset{H}{\rightarrow}-\cos(\omega t) \tag{2}$$

$$s(t)\times x(t)\overset{H}{\rightarrow}s(t)\times\tilde{x}(t) \tag{3}$$

where $s(t)$ is a slower changing function than $x(t)$ and the requirement on signals stated by Eq. (3) is frequently ignored when HT is applied. A single-frequency sinusoid signal $x(t) = c\,\mathrm{e}^{-\xi\omega t}\sin(2\pi\omega t+\theta)$ is chosen with $c=10$, $\xi=0.01$, $\theta=\pi/3$ and $\omega=450$ Hz to show the properties presented in Eqs. (2) and (3). By applying HT, the signal is converted into an analysis signal $\overline{x}(t) = c\,\mathrm{e}^{-\xi\omega t+(2\pi\omega t+\theta)j}$ with $j=\sqrt{-1}$, and then both of the damping ratio ξ and the frequency ω can be extracted, which are illustrated in Figs. 1 and 2, respectively. As shown in Figs. 1 and 2, the frequency and damping ratio are exactly extracted and such a simulation case shows that the property of HT stated by Eq. (3) would work well in the experimental analysis part.

3. Complex ICA with reference for BSS

BSS is a powerful signal processing method and rapidly developed in the late 1980s. As the combination of artificial neural network (ANN), statistics signal processing and information theory, BSS has become an important topic in many research fields with rich potential applications. For linear instantaneous-mixing model, there are separating algorithms based on different cost functions [21,22], second-order statistics or higher order statistics [23,24], non-stationary [25] and other signal features [26,27]. As stated in all of the literatures, no priori knowledge about the mixing matrix and sources is known in advance. And if a priori knowledge on the mixing procedure or sources is introduced, the separation performance would be expected to be better and then constrained ICA [28,29] for BSS or semi-BSS is put forward [30,31]. Recently, complex-valued signals arise frequently in diverse applications of radar, functional magnetic resonance imaging and communications. So real-valued BSS algorithms are extended to process complex-valued signals [32–35]. The linear instantaneous-mixing BSS model is

$$\mathbf{x} = \mathrm{A}\mathbf{s} \tag{4}$$

where $\mathbf{x} = \left(x_1, \ \ldots \ , x_n\right)^T$ is the vector of observed signals, $\mathbf{s} = \left(s_1, \ \ldots \ , s_m\right)^T$ is the source vector and $\mathbf{A}\in C^{m\times n}$ is an unknown constant complex mixing matrix, which should be real in this study as an exception case. For simplicity, we assume $m=n$. The main task of ICA problem for BSS is to estimate a separating matrix $\mathbf{W}$ that yields the independent

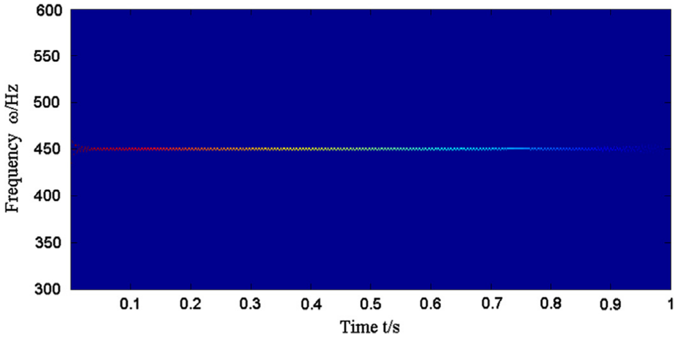

Fig. 1. The HT spectrum of a single-frequency sinusoid signal.