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On the relationship between wave based control, absolute vibration suppression and input shaping

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ABSTRACT

The modeling and control of continuous flexible structures is one of the most challenging problems in control theory. This topic gains more interest with the development of slender space structures, light weight aeronautical components or even traditional gears and drive shafts with flexible properties. Several control schemes are based on the traveling wave approach, rather than the more common modal methods. In this work we investigate the relationships between two of these methods. The Absolute Vibration Suppression (AVS) controller, which was developed for infinite dimension systems, is compared to Wave Based Control (WBC) which was designed primarily for lumped systems. The WBC was first adjusted to continuous systems and then the two controllers, whose algorithms seem different, are compared. The investigation shows that for the flexible shaft these two control laws are actually the same. Furthermore, when converted into an equivalent open loop controller they appear as an extension to continuous systems of the Input Shaping (IS) methodology.

1. Introduction

The problem of controlling continuous flexible structures poses many challenges to the designer, the first of which is the problem of accurately modeling the system with its infinite number of modes. Having such a model, the problem of finding a controller that meets rigid body requirements, while reducing the excitation of the high frequency modes, still remains. This problem is well known and in recent years has received renewed interest with a large number of publications on the subject. There are several approaches to the problem. The first is to approximate the infinite dimension system by a finite approximation, usually Finite Element Method (FEM) and then use standard techniques, e.g. H_∞ [1], to design the controller. While transforming the problem into the familiar and well explored realm of finite dimension linear systems is convenient, the approach has two inherent drawbacks. First, insight into the system is lost and secondly, reasonable accuracy requires very high order models which increase the controller dimension as well. Many, if not most, of the research reports approach the problem from the modal point of view, namely considering the response as a sum of standing waves and controlling it accordingly [2–5]. This approach is geared more towards regulation than tracking, and also suffers from spillover [2] where un-modeled high frequencies may cause instability. A third approach, which is discussed in this paper, is modeling by means of traveling waves and designing a controller from that point of view.

The Absolute Vibration Suppression (AVS) control method stems from modeling second order flexible structures that are governed by the wave equation by infinite dimension transfer function with a distinct structure [6]. This transfer function

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model preserves the physical properties of the system, in particular the traveling wave motion that appears as time delay. The delays arise from the time it takes the wave to travel from the actuation to the measurement point, directly and after reflections, and are completely missing from the modal and FEM models. The notion of transfer functions for infinite dimension flexible dynamic systems appears in prior publications, e.g. [7–9]. However these transfer functions are given as Green's functions, infinite series or generalized matrix exponent in the s domain and have a different structure (except for [7] that considers the free–free case). The first version of an AVS controller was given in [10] but the wave interpretation of the controller was first given in [6] and then investigated in a number of extensions such as multi-link systems [11,12], robustness [13,14] and free response [15]. The AVS controller eliminates the vibrations completely: hence its name.

Wave Based Control (WBC) was developed originally for lumped linear systems [16,17]. It is based on the observation that the actuation of a flexible system at one end will cause a wave to travel through the system and be reflected from the other end to the actuation end. The control action signal is a combination of a launch signal that generates the required motion and a signal that absorbs the returning wave energy based on measurements close to the actuation end. In order to separate the two signals the actual system is virtually connected to an infinite system that absorbs the vibration energy. This virtual system is the basis for the control law. A similar problem is solved independently in [18] by means of virtually extending the system to infinity and choosing appropriate initial conditions for the imaginary system. WBC was used in [16,19] for controlling a flexible robotic arm. In [20,21] the method was extended further to controlling non-linear continuous systems (large deflections of a flexible arm and Euler–Bernoulli beams respectively).

Motion control of flexible structures may also be done in open loop. The main approach, usually referred to as Input Shaping (IS), uses a convolution of a control signal, designed for the rigid body motion, with a series of impulses designed to reduce vibration, thus producing a modified, open loop input signal [22]. The IS methodology was extended in several directions such as minimum time [23] and robustness [24], where the controller design makes allowances for variations in the system parameters (natural frequencies and damping). It is shown that by relaxing the requirement on vibration suppression better robustness is achieved at the cost of prolonging the maneuvering time.

In this paper we compare AVS and WBC, in the case of continuous systems that are governed by the wave equation, and then both of them to IS. Preliminary results in this direction were given in [25]. We begin in Section 2 with presenting the transfer function model of the system. In Section 3 this model is used to construct the AVS control law. In Section 4 the WBC is briefly presented while the main results are given in Section 5. WBC is translated to the continuous setting and then, having a common ground, is compared with AVS. Then the two closed loop controllers are compared with IS. The results of Section 5 are demonstrated by examples in Section 6 and summarized in Section 7.

2. Transfer function model of flexible systems governed by the wave equation

The behavior of some flexible dynamic phenomena, such as transversal motion of a taut string, longitudinal motion of a flexible rod and the angular displacement of a flexible, cylindrical shaft around its axis (see Fig. 1) is described by the one-dimensional wave equation. Assuming homogeneous media and a lumped generalized force at a point $x=x_0$ the dynamics of the system is described by a non-homogeneous wave equation

$$\frac{1}{c^2}\frac{\partial^2 y(x,t)}{\partial x^2}-\frac{\partial^2 y(x,t)}{\partial t^2}=\frac{1}{c\phi}F(t)\delta(x-x_0) \tag{1}$$

where x is the spatial coordinate, y is the flexible displacement, c is the wave propagation velocity in the media, ϕ is a constant that depends on the material and the geometry and F is the general force operating at x_0. For example, in a flexible

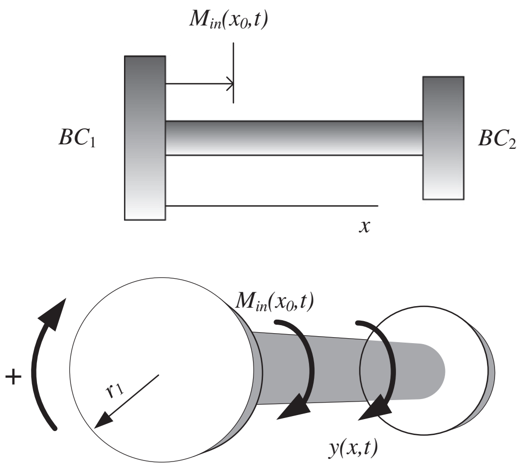

Fig. 1. Single flexible shaft.