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Boundary-controlled travelling and standing waves in cascaded lumped systems

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ABSTRACT

This paper shows how pure travelling waves in cascaded, lumped, uniform, mass–spring systems can be defined, established, and maintained, by controlling two boundary actuators, one at each end. In most cases the control system for each actuator requires identifying and measuring the notional component waves propagating in opposite directions at the actuator–system interfaces. These measured component waves are then used to form the control inputs to the actuators. The paper also shows how the boundaries can be actively controlled to establish and maintain standing waves of arbitrary standing wave ratio, including those corresponding to the classical modes of vibration of such systems with textbook boundary conditions. These vibration modes are achieved and maintained by controlled reflection of the pure travelling wave components. The proposed control systems are also robust to system disturbances: they react to overcome external disturbances quickly and so to re-establish the desired steady motion.

1. Introduction: the problem

This paper considers the creation and maintaining of wave-like motion in systems consisting of strings of lumped masses and springs, connected in series, driven by two actuators, one at each end, such as in Fig. 1. A recent paper [1] shows that the minimum number of actuators to achieve pure travelling waves in a uniform, distributed, rectilinear system is two, one at each boundary. This is an intuitive result. Fig. 1 shows the corresponding lumped system. It is assumed that two boundary actuators are necessary and sufficient also in the lumped case. The problem then is how to control these two actuators first to establish, and then to maintain, any desired, physically possible, travelling wave, or standing wave (vibration mode), or standing wave ratio, within the lumped, cascaded system.

Regarding the actuation, note that if, instead of having only two, boundary actuators, each mass has more direct, controlled actuation, then arbitrary, wave-like motion along the system can be achieved by suitably moving each discrete mass to create the desired overall wave effect, whether propagating or standing. In this case any dynamic coupling between the masses becomes secondary, indeed dispensable. The control problem is then primarily a kinematic one, with each actuator moving locally to create any desired overall wavelength, wave frequency, wave speed and waveform along the system.

But, in this work, the only controlled actuation is at the boundaries. The springs in the cascade provide the coupling of motion between the masses. The actuator control systems must then understand, respect and exploit these mass–spring dynamics and the way motion propagates through them.

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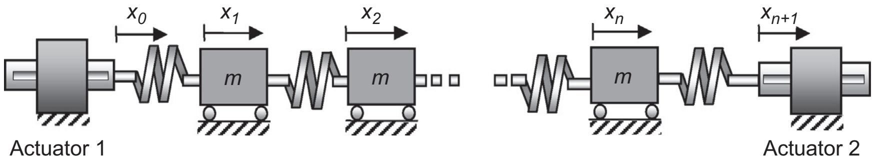

Fig. 1. A lumped mass–spring string controlled by two, position-controlled, boundary actuators.

At first sight the problem seems simple enough. For example, suppose it is desired to set up an harmonic travelling wave, of a specified frequency, propagating along the cascade of Fig. 1, beginning at one actuator and ending at the other. Then, at steady state, clearly the two actuators must move harmonically, with a common amplitude and frequency, and a time delay (or phase lag) between them corresponding to the wave travel time along the system. That much is clear and apparently simple.

But a number of challenges immediately arise. First of all, what is the appropriate phase lag between the two actuators? In other words, what is the wave travel time at the specified frequency? For such a lumped system the wave speed (assuming it can be clearly defined) is certainly frequency dependent. The system is also inherently dispersive, even assuming ideal behaviour and perfectly known parameters, so in general waves will distort as they travel, more obviously in the case of transients.

Secondly, how does one reach steady state? If the system is starting from any initial state, whether known (at rest, say) or unknown, even if the correct, steady-state, phase delay is known beforehand, simply setting the two actuators to move with the correct phase delay will not achieve steady motion. Transients will certainly arise and they can be of large amplitude and quite anarchic. If damping is negligible the "transients" (so-called) could endure indefinitely, spoiling the desired travelling waves. If damping is significant, the transients might (one hopes) die out, but now the travelling wave will also be attenuated as it propagates. So in addition to the phase differences, the amplitudes of the two actuators should no longer be equal if a pure, attenuated, travelling wave is to be created, but it is not clear what these amplitude and phase differences should be. Thus, with or without damping, there are unresolved practical and theoretical difficulties.

If setting up a desired travelling wave is challenging, it proves no easier to establish *standing* waves: in other words, waves of equal magnitude and frequency propagating in opposite directions, which when superposed create standing waves, typically with nodes and antinodes at fixed points along the system. A more general problem is how to set up a specified intermediate situation where there is a "standing wave ratio", characterised by amplitude variations whose envelope forms a fixed spatial pattern. This situation can be interpreted as a travelling wave component combined with a standing wave component, arising from wave reflection and absorption conditions at the actuators, or as the resultant of two waves of equal frequency but unequal magnitude travelling in opposite directions in the system, producing the standing pattern.

The standard, textbook approach to modelling cascaded lumped systems is modal analysis. In modal analysis, boundary conditions are specified, whether dynamic (also called "natural"), kinematic ("geometric"), or mixed. Resonance frequencies and mode shapes are then determined by assuming synchronous, steady-state motion. But what if, instead of doing a theoretical exercise, one is attempting to control a physical system such as Fig. 1, and it is desired to get the actuators to reproduce such specific vibratory responses? This is now a practical control question. The actuators can be taken to correspond to the system boundaries of modal analysis. But how should their controllers work to establish a desired mode (starting from rest, for example), and then how should they maintain it (despite inevitable minor disturbances)? The actuators are required to get the system to a steady synchronous motion, which includes having the actuators reproduce the specified boundary conditions (which, paradoxically, might include fixed boundaries). But before reaching and maintaining the specified steady state, the actuators will need to pass through a very specific, controlled, start-up transient, and it is not obvious how to specify this. Again this apparently simple, practical problem proves quite challenging and raises non-trivial questions.

A further question considered in this paper is how to simulate more general boundary conditions, where one of the actuators can simulate the interaction of the given cascaded system with another dynamic system with its own dynamic response (other than simple reflection or absorption). This is a generalisation of the "natural" boundary conditions of modal analysis.

The present paper presents novel solutions to all of these challenges. For simplicity the following assumptions will be made. (a) Only uniform systems (with all masses and all springs of equal value) will be considered. (b) The actuators will be assumed to be ideal, with output motion equal to the requested input. (c) Damping, both internal (between masses) and external (masses to ground) will be assumed to be negligible.

None of these assumptions is essential. The work can be extended to non-uniform systems, systems with real actuators, and systems with damping. These assumptions are chosen simply to reduce the variety of issues to be treated and so limit the length of the paper. The other cases will be considered separately.

Before presenting the method, it is worth briefly considering if solutions to these challenges already exist, and why one might be interested.