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Bounds for the stationary stochastic response of truss structures with uncertain-but-bounded parameters

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ABSTRACT

The aim of the present paper is to determine the region of the probabilistic characteristics of the stationary stochastic response (mean-value vector, power spectral density function and covariance matrix) of truss structures with uncertain-but-bounded parameters under stationary multi-correlated Gaussian random excitation via *interval analysis*. The main steps of the proposed procedure are: i) to express the stiffness, damping and mass matrices of the structural system as linear functions of the uncertain-but-bounded parameters; ii) to split the probabilistic characteristics of the nodal interval stationary stochastic response, evaluated in the frequency domain, as sum of the midpoint and deviation values; iii) to evaluate in explicit approximate form the parametric interval frequency response function matrix. The effectiveness of the presented procedure is demonstrated by analyzing a truss structure with uncertain-but-bounded axial stiffness and lumped masses subjected to stationary multi-correlated Gaussian wind excitations.

1. Introduction

The main dynamic excitations arising from natural phenomena, such as earthquake ground motion, gusty winds or sea waves, are commonly modeled as Gaussian stochastic processes for structural analysis purposes. In the framework of Stochastic Mechanics, several approaches have been proposed to cope with the challenging problem of characterizing the random response of a structural system under stochastic excitation. In particular, it is well-known that the random response is fully defined from a probabilistic point of view by the knowledge of its probability density function (PDF). Moreover, if the system has a linear behavior and it is forced by a Gaussian random process, the response is Gaussian too. In this case, the probabilistic characterization of the stochastic response can be performed either in the so-called *time domain* by evaluating the mean-value vector and the correlation function matrix, or in the so-called *frequency domain* through the knowledge of the mean-value vector and the power spectral density (PSD) function matrix (see e.g. [1,2]).

Another class of uncertainties occurring in engineering problems is the one associated with fluctuations of structural parameters, such as geometrical and mechanical properties or mass density. These sources of uncertainty, which affect to a certain extent the structural response, are usually described following two contrasting points of view, known as probabilistic and non-probabilistic approaches. In the framework of the so-called probabilistic approaches, the uncertain structural parameters are modeled as random variables with given PDF. The associated random structural response is

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usually predicted following three main ways: the Monte Carlo simulation method, the stochastic finite element method [3] and the spectral approach [4]. A discussion on the application of probabilistic approaches to the case of truss structures can be found in Ref. [5,6].

Despite their success, unfortunately the probabilistic approaches give reliable results only when sufficient experimental data are available to define the PDF of the fluctuating properties. If available information are fragmentary or incomplete so that only bounds on the magnitude of the uncertain structural parameters are known, non-probabilistic approaches, such as convex models, fuzzy set theory or interval models, can be alternatively applied [7]. The interval model, which stems from *interval analysis* (see e.g. [8–11]), may be considered as the most widely used analytical tool among non-probabilistic methods. According to this approach, the fluctuating structural parameters are treated as interval numbers with given lower and upper bounds. The application of the interval analysis method to practical engineering problems is not an easy task due to the complexity of the related algorithms. In the literature, in the case of slight parameter fluctuations and deterministic loads, the so-called *Interval Perturbation Method* (*IPM*) or, equivalently, the *First-Order Interval Taylor Series Expansion* have been successfully adopted to perform both static [12–16] and dynamic structural analysis [17–19]. The main advantages of these methods are the flexibility and the simplicity of the mathematical formulation.

In the framework of Stochastic Mechanics, the *IPM* has been extended by the authors [20,21] to cope also with randomly excited structures. However, since the effectiveness of perturbation-based approaches is limited to small parameter fluctuations and the effect of neglecting the higher-order terms is unpredictable, recently the authors proposed an alternative approach, to characterize, in the time domain, the stochastic response due to both stationary [22] or non-stationary [23,24] random excitations. Notably, in these papers, the drawbacks associated with the so-called *dependency phenomenon* are overcome. This phenomenon [11,25,26] frequently occurs in the “ordinary” interval analysis when an expression contains multiple instances of one or more interval variables. Indeed, the “ordinary” interval analysis, mainly based on the formulation derived by Moore [8], often leads to an overestimation of the interval solution width that could be catastrophic from an engineering point of view. This happens when the operands are partially dependent on each other so that not all combinations of values in the given intervals will be valid and the exact interval will be smaller than the one produced by the formulas.

Interval-based uncertainty models have been extensively used in the context of static (see e.g. [27–29]) and dynamic (see e.g. [19,30,31]) finite element analysis of structures. A general overview of the state-of-art and recent advances in interval finite element analysis can be found in Ref. [26,32], where the two fundamental approaches are described in details: the interval arithmetic strategy and the global optimization approach. The application of interval arithmetic based finite element methods is hindered by the dependency phenomenon which introduces conservatism not only in the solution phase, but also in the assembly of the system matrices. In the literature, several attempts have been made to limit conservatism, such as the element-by-element technique developed by Muhanna and Mullen [25] or the improvement of interval finite element analysis proposed by Degrauwe et al. [33] based on affine arithmetic.

As far as the case of random input is concerned, in all the studies carried out by the authors the uncertain-but-bounded stochastic response is characterized in the time domain by solving a set of linear algebraic equations or a set of first-order differential equations, depending on whether the excitation is stationary or non-stationary, respectively. Both these sets of equations are derived by applying the Kronecker algebra [34]. The main computational drawback of the time domain formulation is associated with the evaluation of the input-output cross-correlation appearing in the forcing term, which can be determined in closed-form only for white or filtered white stochastic input processes.

The aim of the present paper is to determine the region of the random response of truss structures with uncertain-but-bounded parameters under stationary multi-correlated Gaussian stochastic excitation by applying the *complex interval analysis* [9] within the context of the *frequency domain approach*. Unfortunately, the “ordinary” complex interval analysis suffers from the dependency phenomenon too. In the framework of the so-called *affine arithmetic* [35,36], Manson [37] proposed an approach which allows to take into account the dependency between the real and imaginary components of the complex interval variables. In the present paper, the catastrophic effects of the dependency phenomenon are limited by adopting an improvement of the ordinary complex interval analysis, based on the philosophy of the affine arithmetic. The proposed procedure basically requires the following steps: i) the decomposition of the stiffness, damping and mass matrices of the structural system as sum of the nominal value plus a deviation given as a linear function of the uncertain-but-bounded parameters; ii) the evaluation in explicit approximate form of the parametric interval transfer function matrix, by applying the so-called *rational series expansion* recently proposed by Muscolino et al. [38], and of the inverse of the interval stiffness matrix by using the *extended interval-valued Sherman–Morrison–Woodbury formula* derived by Impollonia and Muscolino [39]; iii) the determination of the upper and lower bounds of the probabilistic characteristics of the nodal stationary stochastic interval response (mean-value vector, power spectral density functions and covariance matrix).

Numerical results concerning a truss structure with uncertain-but-bounded axial stiffness and lumped masses under stationary multi-correlated Gaussian wind excitations are presented to show the effectiveness of the proposed method.

2. Problem formulation

2.1. Preliminary definitions: real and complex interval variables

In this paper, the attention is focused on truss structures with uncertain-but-bounded structural properties under stationary Gaussian random excitation. Following a standard formulation, the stiffness, damping and mass matrices of a