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High-resolution direction finding of non-stationary signals using matching pursuit

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ABSTRACT

One of the main goals of time–frequency (TF) signal representations in non-stationary array processing is to equip multi-antenna receivers with the ability to separate sources in the TF domain prior to direction finding. This permits high-resolution direction-of-arrival (DOA) estimation of individual sources and of more sources than sensors. In this paper, we use linear decomposition of sensor data based on matching pursuit (MP). The leading atoms of the MP, which capture most of the source TF signatures, can be different for different sources and, as such, provide the desired source discrimination. The MP coefficients with high signal-to-noise ratio (SNR) and corresponding to the leading decomposition atoms are used to develop the MP-MUSIC DOA estimation for non-stationary source signals. We demonstrate the source discriminatory capability of the proposed technique using linear FM, nonlinear FM, and other non-stationary signals. Further, we compare MP-MUSIC performance with conventional MUSIC and the time–frequency MUSIC, which incorporates bilinear transforms.

1. Introduction

High-resolution direction finding of non-stationary signals can exploit the time–frequency (TF) signatures of the sources in the field of view to provide source discrimination and increased signal-to-noise ratio (SNR) [1]. Both capabilities have been achieved within the spatial time–frequency distribution (STFD) framework. This framework was applied to narrowband signals in [2,3] and extended to wideband sources by Gershman et al. in [4–6]. The STFD framework applies a form of joint-variable signal representations to expose hidden TF signatures characterizing the data received by the antenna array. Signal analysis in a single domain, whether time or frequency, fails to reveal the local behavior of the signal and in expressing its power distribution over both time and frequency. On the other hand, bilinear transforms, such as Cohen's class [7] of time–frequency distributions (TFD), capture the instantaneous frequency (IF) laws underlying the non-stationarity of the data.

The STFD matrix, in lieu of the covariance matrix, permits the auto- and cross-TFDs of the sensor data to retain the signal phase and, as such, embeds the sources' direction-of-arrival (DOA) information. DOA estimation approaches using subspace methods, such as MUSIC [8], and incorporating the STFDs have been shown to improve the performance over their covariance matrix counterparts, primarily because of their capability to successfully discriminate among sources and exclude some from consideration prior to subspace decomposition. Accordingly, the STFD-based DOA approaches become attractive for sources with close angular separations, but with distinctive IFs [9].

In this paper, we provide an alternative to the STFD framework which enjoys the same benefits of STFDs. We use

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linear, in lieu of quadratic, TF signal representations by employing matching pursuit (MP) [10] in which the decomposition coefficients bear the source TF localization profiles. These coefficients act like the signal auto-terms in bilinear TF transforms. However, unlike the bilinear STFD approach, where the cross-sensor distribution is needed to capture the phase changes across the array, the linear decomposition of the data at each sensor using MP preserves the signal phase. The MP-MUSIC is developed by forming the coefficient covariance matrix and then applying eigendecomposition for subspace estimation.

MP is an adaptive signal decomposition technique that is energy conservative. While first introduced by using the Gabor functions as atoms, MP has been extended to use dictionaries consisting of any Gaussian envelopes with arbitrary phase laws (e.g., constant, linear, cubic and polynomial). Clearly, using atoms with more parameters provides higher flexibility in matching the signals, but also increases the computational cost. From DOA estimation perspective, MP offers the same key advantages of STFD, i.e., source discrimination and SNR enhancement. It is applicable to a broad class of non-stationary signals, not necessarily those characterized by their IFs. Further, unlike the STFD, where TF points or regions of high power concentrations need to be identified post distribution computations through thresholding, MP automatically and chronologically identifies atoms that capture these regions according to their energy contributions. This directly determines the best TF regions to be incorporated in DOA estimation.

MP was used to estimate the source DOA in a manner similar to the maximum likelihood (ML) method in [11–13]. In these approaches, DOA estimation is performed by utilizing a large dictionary that includes steering vectors for all possible signal arrivals. Although the algorithm converges over few snapshots, the computational cost is rather considerable. This procedure of applying MP for steering vectors is entirely different from the one proposed in this paper, where the source non-stationarity is addressed by the MP decomposition in the temporal domain, followed by subspace decomposition of the MP coefficient covariance matrix for DOA estimation. Linear TF decompositions, using the wavelet transform, have been used for de-noising prior to DOA estimation in [14–17]. In contrast to the wavelet approach, the proposed approach incorporates the MP into DOA and uses the decomposition coefficients directly into signal and noise subspace decompositions.

In this paper, we develop high-resolution DOA estimation techniques of non-stationary narrowband signals using MP, and demonstrate the proposed technique's source discriminatory capability and its robustness against noise. A priori knowledge of the coarse source TF behavior, if available, can aid in tailoring the atoms to a specific problem. This knowledge can be gained, e.g., from the TFD of the reference sensor or from the averaged TFDs across the sensor array (e.g., [18]). In the absence of this knowledge, the chirplet atoms can be adopted because of their attractive TF concentration properties [19–22]. When proper matching of atoms and signals occurs, each of the leading atoms captures one signal, allowing DOA estimation of a single source to be performed for each atom. The consequence of using a general set of atoms, like chirplets, in the absence of *a priori* information of the signals is twofold. First, more atoms are required to properly decompose the signals and capture their energy. Second, some atoms may overlap with two or more sources. These atoms assume a similar role of cross-terms in the STFD framework and may hinder source discrimination. In both cases, an association procedure is performed to attempt to group the atoms and the coefficients [23].

This paper is organized as follows. In Section 2, we introduce the signal model, and review the conventional MUSIC technique as well as the STFD concept. A short overview of the MP decomposition is provided in Section 3. In Section 4, we develop MP-MUSIC for DOA estimations. It is shown that the MP coefficient covariance matrix can be used as an alternative to the spatial covariance and STFD matrices to formulate subspace-based DOA estimation methods within the MP framework. The MP-MUSIC performance is provided in Section 5. Section 6 presents simulation results, and finally Section 7 concludes this paper.

The following notations are used in this paper. Boldface lower-case letters (e.g., $\mathbf{a}$) denote vectors, and boldface upper-case letters (e.g., $\mathbf{A}$) denote matrices. E[.] represents the statistical mean operation. $(.)^*$, $(.)^T$ and $(.)^H$ denote complex conjugate, transpose and conjugate transpose, respectively. $\delta(.)$ denotes the Kronecker delta function, and $\mathbf{I}$ is an identity matrix. In addition, $C^{M\times N}$ denotes the space of $M\times N$ matrices with complex entries. $<\mathbf{a},\mathbf{b}> = \mathbf{a}^T\mathbf{b}^*$ denotes the inner product of two vectors $\mathbf{a}$ and $\mathbf{b}$, and $\|.\|$ denotes the Frobenius norm of a vector.

2. Signal model

In this section, we introduce the signal model. For the convenience of presentation, we briefly review the MUSIC and the STFD concept [1,8].

2.1. Signal model

Assume K non-coherent narrow-band signal sources impinging on an M-element with angles θ_k, $k=1, 2,\ldots, K$. The array output vector at time instant t, $\mathbf{x}(t)=[x_1(t), x_2(t),\ldots,x_M(t)]^T$, is expressed as

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t),\quad t=1, 2,\ldots, N \tag{1}$$

where N is the number of snapshots, $\mathbf{s}(t)=[s_1(t), s_2(t),\ldots, s_K(t)]^T$ is the source signal vector, and $\mathbf{n}(t)=[n_1(t), n_2(t),\ldots,n_M(t)]^T$ is an additive noise vector whose elements are modeled as stationary, spatially and temporally white Gaussian, zero-mean complex random processes, independent of the source signals, i.e., $E[\mathbf{n}(t+\tau)\mathbf{n}^H(t)]=\sigma_n^2\delta(\tau)\mathbf{I}$, with σ_n^2 denoting the variance. In particular, when the array is uniform linear, then the kth column of the steering matrix $\mathbf{A}=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\ldots,\mathbf{a}(\theta_K)]$ is expressed as $\mathbf{a}(\theta_k)=[1,e^{j\omega_k},\ldots,e^{j(M-1)\omega_k}]^T$, where $\omega_k=2\pi(d/\lambda)\sin(\theta_k)$ is the spatial frequency of the kth signal, λ denotes the wavelength, and d is the inter-element spacing.

The spatial covariance matrix of $\mathbf{x}(t)$ is defined as

$$\mathbf{C_{xx}}=E[\mathbf{x}(t)\mathbf{x}^H(t)]=\mathbf{A}\mathbf{C_{ss}}\mathbf{A}^H+\sigma_n^2\mathbf{I}, \tag{2}$$