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# Probabilistic model of the human cortical bone with mechanical alterations in ultrasonic range

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ABSTRACT

The biomechanical materials are among the most complex mechanical systems. Most often, their microstructure is heterogeneous and random. This is the case for the human cortical bones which are considered in this paper. For such systems, a gradient of porosity can be observed in the thickness direction but, in this case, none of the usual theories of porous materials can be applied. For this reason, a simplified model with gradient for the elasticity tensor is presented. The predictability of this model is improved by taking into account uncertainties. We propose a prior stochastic model of the tensor-valued elasticity field corresponding to an extension of a previous work in which the random elasticity field was constant in space. This extension consists in introducing two ingredients: the introduction of a spatial gradient for the mean elasticity tensor and spatial statistical fluctuations in the thickness direction. The stochastic model which is constructed shows that the observed responses are effectively sensitive with respect to the values of the gradient and to the level of statistical fluctuations. In this sense, this stochastic model will be well adapted to perform its identification solving a stochastic inverse problem that is not the purpose of the present paper.



## 1. Introduction

The aim of this paper is to introduce a stochastic elastoacoustic model useful for the ultrasonic characterization of a multilayer materials for which an elastic solid layer is uncertain. This stochastic model is developed in the context of the ultrasonic propagation in human cortical bones but could be applied to another mechanical systems. The biomechanical materials are among the most complex mechanical systems. Modeling such media is a challenge and the main difficulty is given rise to the complexity level of their microstructures. This is the case for the human cortical bones which are considered in this paper. The complexity level of such a biomechanical system is such that a multiscale approach should be developed to represent all the mechanical behavior and would allow (among others) the ultrasonic characterization of a human compact bone to be simulated. Nevertheless, such a multiscale model would be very difficult to construct taking into account the complexity of the heterogeneities of the microstructure. In this paper, we are only interested in the ultrasonic characterization of the human cortical bone. In this context, a first simplified model has been introduced in [16,9] and which is a representative of *in vivo* measurements of bone cortical properties with the so-called axial transmission technique [13]. This simplified elastoacoustic model is a three-layer system: an elastic solid layer sandwiched between two acoustic

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fluid layers. The elastic solid layer (cortical bone) is an anisotropic elastic material, while the fluid acoustic layers (soft tissues) are usual compressible and homogenous inviscid fluids satisfying the linear acoustic equations. The axial transmission technique provides the velocity of ultrasonic waves axially transmitted along the cortical bone through a linear arrangement of transmitters and receivers placed on the same side of the skeletal site. This technique has been extensively used to probe bone quality at the radius [4,8]. With the bidirectional axial transmission technique, the measurements of the velocity of the first arriving signal (FAS) are sensitive to the elasticity properties of the cortical bone in the neighborhood of the coupling interface between the elastic solid layer (cortical bone) and the acoustic fluid layer (soft tissues). Nevertheless, previous works [5,6,10,17] show that the FAS is not sensitive to the bone mass density and to the bone thickness when this one is greater than the wavelength in the cortical bone. This is a reason why, in this paper, an extension of the stochastic model and an additional observation are introduced in order to also obtain a sensitivity with respect to the bone thickness. It should be noted that the *in vivo* measurements [9] exhibit random fluctuations of the ultrasonic characterization which are not related to measurement errors induced by the probe. In Desceliers et al. [9], it was seen that a simplified mechanical model with an additional stochastic modeling of bone elasticity properties are able to represent the *in vivo* measurements in the statistical sense. It should be noted that the first application of the parametric probabilistic approach to the axial transmission technique can be found in Macocco et al. [14].

Besides its multiscale nature, cortical bone is heterogeneous at the organ scale. Its mechanical properties depend on the cross-sectional and axial location [19,20]. This heterogeneity depends on properties such as porosity and three degrees of mineralization of bone tissues [18]. In particular, porosity in the radial direction (which is associated with the cross-section) is heterogeneous at all ages and for both genders [7]. The probabilistic model introduced in this paper corresponds to a mesoscale stochastic modeling of the microstructure because at the micro-scale, the microstructure cannot be described in terms of constituents. In this paper, for such a biomechanical system, we consider that the microstructure of the cortical bone is altered in the neighborhood of its interface with the marrow. As a result, strong fluctuations of the pore sizes can exist in the thickness direction but, in this case, none of the usual theories of porous materials [1–3] can be applied because the variations of the pore sizes are too much important with respect to the number of pores and consequently, homogenization cannot be applied. Consequently, in the thickness direction, the spatial variation of the mean elasticity properties for the mean model at the mesoscale modeling of the microstructure is taken into account in introducing a gradient of the elasticity properties. The determination of the gradient of the mean elasticity properties is of a first interest in order to diagnose bone diseases through the quantification of bone stiffness. Thus, inside the elastic solid layer (cortical bone), the constitutive equation goes to one of the acoustic fluids (marrow) when one moves towards the marrow in the thickness direction. The uncertainties related to such a model are taken into account by modeling the elasticity tensor by a random field for which the mean value corresponds to the previous mean model exhibiting a gradient.

In the following section, the simplified mean model introduced in Desceliers et al. [9] is briefly recalled. The next three sections are devoted to the construction of the gradient model for the mean elasticity properties of the microstructure at the mesoscale. In the following section, the stochastic model around the mean model is constructed. Finally, we introduce a new observation allowing the determination of the parameters of the probabilistic model to be performed and we present a numerical illustration for the cortical bone.

## 2. Simplified model

The properties of the human cortical bone are studied by using *in vivo* measurements obtained with the axial transmission technique. An acoustic pulse is applied to the skin layer in the ultrasonic range and the velocity of the first arriving signal is measured. A simplified model of the human cortical bone made of the skin, the coupling gel with a probe which generated an acoustic pulse and the marrow has been developed in Naili et al. [16] and Desceliers et al. [9]. This simplified model is composed of an elastic solid semi-infinite layer between two acoustic fluid semi-infinite layers as shown in Fig. 1.

Let $\mathbf{R}(O,\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ be the reference Cartesian frame, where $O$ is the origin of the space and $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ is an orthonormal basis. The generic point in $\mathbb{R}^3$ is $\mathbf{x}=(x_1,x_2,x_3)$. The thicknesses of the layers are denoted by $h_1$, $h$ and $h_2$ (see Fig. 1). The first

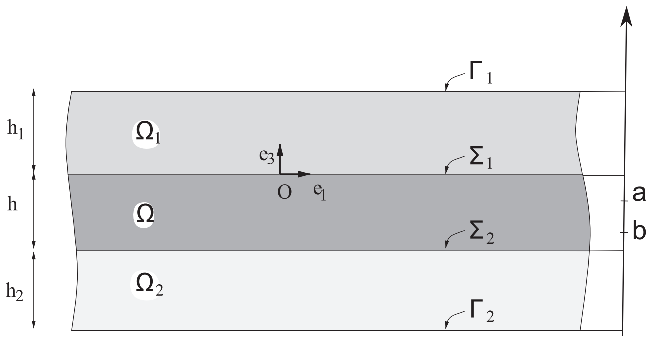


**Fig. 1.** Geometry of the multilayer system.