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Pure-rotary periodic motions of a planar two-ball auto-balancer system

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ABSTRACT

Ball-type automatic balancers have been widely used to suppress the unbalanced vibration of rotor systems. However, instead of reaching the desired perfect balancing position, where the balls of the automatic balancer are allocated properly so that the rotor is perfectly balanced, the system may settle into a pure-rotary periodic motion, in which all the balls stick together and keep rotating around the balancer. Because the associated large vibrations may deteriorate the performance of the rotor system, it is desirable to avoid the pure-rotary periodic motion. To this end, there is a need to understand the properties of pure-rotary periodic motions clearly. In this study, we used the modified incremental harmonic balance method to find pure-rotary periodic motions numerically. The existence and stable regions of the pure-rotary periodic motion in a two-parameter plane were identified. The effects of system parameters on the stable regions of the pure-rotary periodic motion were examined. By comparing the stable regions of the pure-rotary periodic motion with those of the perfect balancing position, the variation of the steady-state response with the rotational speed was investigated. We also conducted experiments to test the stability of the pure-rotary periodic motion under different conditions. The experimental results agree well with the numerical results.

1. Introduction

Ball-type automatic balancers are effective for suppressing rotational vibrations due to inherent unbalance of the rotor and have been applied in different fields, e.g. optical disk drives [1–6] and hand-held tools [7]. Such a ball-type automatic balancer consists of several balls moving along a circular orbit. Under proper conditions, the balls will move in specific positions and balance the rotor perfectly. This particular equilibrium configuration will be referred to as the perfect balancing position henceforth. For an automatic balancer to be useful, the perfect balancing position needs to be stable under the working conditions of the rotor. But even when it is the case, the system may still settle into different types of periodic motions and generate relatively large vibrations. In a periodic motion, each ball may either rotate around the balancer or oscillate about a mean position. In this paper, for the sake of convenience of discussion, the periodic motions of a two-ball automatic balancer are classified into three different types according to the behavior of the balls along the circular orbit:

(i) Pure-oscillatory periodic motion—each ball oscillates about a mean position.
(ii) Pure-rotary periodic motion—the two balls stick together and rotate around the center of the orbit.

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(iii) Compound-rotary periodic motion—one ball rotates around the center of the orbit while the other ball either rotates around the center of the orbit with a different speed or oscillates about a mean position.

Among these three types of periodic motions, the pure-rotary type is the most important because it occurs much more frequently than the other two types. Pure-rotary periodic motions result in large vibrations and hence may significantly deteriorate the performance of the rotor system. To prevent the system from settling into this undesired motion, a comprehensive understanding of the pure-rotary periodic motion is in order.

Much research has been devoted to the investigation of the equilibrium solutions of a planar auto-balancer system consisting of a rotating disk and a ball-type automatic balancer [8–10]. The stable regions of the perfect balancing position in the design parameter plane were identified [1,4,11–13]. The effects of several factors, e.g., non-planar motion [3,5,14–17], dry friction between the balls and orbit [18–19], and the number of balls [20–21], on the performance of the auto-balancer were examined. Rajalingham and Bhat [14] pointed out that, even though the perfect balancing position is stable, the system may settle down to the pure-rotary periodic motion. The co-existence of multiple stable solutions indicates that a global analysis is needed for a thorough understanding of the dynamic characteristics of the auto-balancer system. Green et al. [22] conducted a detailed bifurcation analysis and found that an auto-balancer system may possess stable equilibrium positions, periodic motions, and even chaotic attractors. In addition to the pure-rotary periodic motion, they have also reported the pure-oscillatory and compound-rotary periodic motions. It is worth noting that, as pointed out by Green et al. [22], while equilibrium positions and pure-oscillatory periodic motions can be easily captured by the continuation package AUTO, the pure-rotary and compound-rotary periodic motions can only be determined by brute force integration. A common feature of the pure-rotary and compound-rotary periodic motions is that at least one ball keeps rotating around the balancer. This type of periodic motions, i.e. the union of the pure-rotary and compound-rotary periodic motions, will be referred to as rotary periodic motions for brevity henceforth. The lack of effective numerical methods may be the reason why the rotary periodic motion of an auto-balancer system has not been investigated thoroughly.

In a rotary periodic motion, at least one ball keeps circulating along the whole orbit. Consequently, the angular coordinate defining the position of the circulating ball relative to the disk increases with time. In this case, although important physical properties of the system (e.g. Cartesian coordinates of the center of the disk) are periodic, the angular coordinate is non-periodic. Specifically, a rotary periodic motion is physically periodic but mathematically non-periodic. In contrast, a pure-oscillatory periodic motion is both physically and mathematically periodic. It is the mathematical aperiodicity that prevents the rotary periodic motion from being determined directly by existing numerical methods, which are limited to detecting mathematically periodic motions. For example, the incremental harmonic balancing (IHB) method is a power method to determine pure-oscillatory periodic motions [23–32]. However, only with proper modifications can this method be used to determine rotary periodic motions. In this paper, we used the modified IHB method developed in [33] to determine the periodic motions of the auto-balancer system.

This paper aims to study the periodic motions of a planar two-ball auto-balancer system both numerically and experimentally. The governing equations of the auto-balancer system were derived using Lagrange's equations. The modified IHB method was briefly introduced and then used to detect the periodic motions. The main focus was on the properties of pure-rotary periodic motions. The stability of the pure-rotary periodic motion was determined by the eigenvalues of the associated monodromy matrix. Stable regions of the pure-rotary periodic motion in a two-parameter plane were identified and compared with those of the perfect balancing position. Experiments were conducted to validate the numerical results. Finally, the variation of the steady-state response with the rotational speed was discussed and the results were summarized.

2. Mathematical modeling and governing equations

Fig. 1 shows the schematic of the system considered and the reference frames. The system consists of a rotating unbalanced disk, a ball-type automatic balancer, and a suspension system. The disk with mass m_d rotates with a constant angular speed ω. The mass center G of the disk is located at a distance e from the geometric center C and results in an amount of unbalance of $m_d e$. The automatic balancer is attached concentrically to the disk and is composed of an orbit containing two balls and a damping fluid with damping constant c_b. The balls, each of mass m_b, can move along the orbit and their movements are subjected to damping force. Let δ denote the radius of the orbit. Then the unbalance caused by a single ball is $m_b\delta$. The disk is supported by an isotropic suspension system that is characterized by linear springs with spring constant k and viscous dampers with damping constant c. All movement is assumed to be confined to the horizontal (X–Y) plane and is not affected by gravity. When the suspension springs are undeflected, the geometric center C of the disk is located at the origin O of the inertial reference frame OXY. The reference frame Oxy rotates with the same speed ω as the disk with respect to OXY and the x-axis is always parallel to CG. The position of the disk is indicated by the coordinates (x,y) of the geometric center C relative to the Oxy frame and the position of the ith ball is given by the angle β_i relative to the mass center G.