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Symmetric self-Hilbertian filters via extended zero-pinning

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ABSTRACT

A symmetric self-Hilbertian filter is a product filter that can be used to construct orthonormal Hilbert-pair of wavelets for the dual-tree complex wavelet transform. Previously reported techniques for its design does not allow control of the filter's frequency response sharpness. The Zero-Pinning (ZP) technique is a simple and versatile way to design orthonormal wavelet filters. ZP allows the shaping of the frequency response of the wavelet filter by strategically pinning some of the zeros of the parametric Bernstein polynomial. The non-zero Bernstein parameters, α_i's, are the free-parameters and are constrained in number to be twice the number of pinned zeros in ZP. An extension to ZP is presented here where the number of free-parameters is greater than twice the number of pinned zeros. This paper will show how the extended ZP can be used to the design of Hilbert pairs with the ability to shape the filter response.

1. Introduction and preliminaries

The critically sampled (non-redundant) Discrete Wavelet Transform (DWT) is a powerful signal processing tool and has found success in many applications like image compression [1], signal denoising [2] and watermarking [3]. Despite its many success the DWT is not without disadvantages and the most important one is that it is shift-invariant due to the multirate nature of the transform. This has lead researchers to consider redundant transforms such as the shift-invariant Discrete Wavelet Transform (SI-DWT) [4] and the higher-density wavelet transform [5]. The dual-tree complex wavelet transform (DT-CWT) of Kingsbury [6] however has emerged recently as one of the most popular redundant transform. The DT-CWT has the advantages of being approximately shift-invariant and providing directional selectivity in multidimensions [7]. The DT-CWT can be implemented efficiently without a significant increase of complexity compared to the critically sampled DWT and complex arithmetic is not needed. The DT-CWT has showed it superiority over the DWT in many applications like denoising [2] and image modeling [8].

The DT-CWT is based on a pair of filter banks whose equivalent wavelet functions are Hilbert transforms of each others:

$$\Psi^g(\omega)=\begin{cases}-j\Psi^h(\omega) & \text{for } \omega>0\\ j\Psi^h(\omega) & \text{for } \omega<0\end{cases} \tag{1}$$

where $\Psi^h(\omega)$ and $\Psi^g(\omega)$ are respectively the Fourier transforms of the wavelet functions $\psi^h(t)$ and $\psi^g(t)$. This paper will focus on orthonormal wavelets and the corresponding conjugate quadrature filter (CQF) banks. The corresponding low-pass filter is denoted as $H^h(z)$ and $H^g(z)$. The necessary and sufficient condition for the wavelets from a pair of CQF $(H^h(z),H^g(z))$ to satisfy (1) is

$$H^g(e^{j\omega})=e^{-j\omega/2}e^{j2\,d\omega}H^h(e^{j\omega}) \quad -\pi\le\omega\le\pi \tag{2}$$

where d is an integer. The half sample delay was first discussed in [9] and subsequently generalized in [10,11] to include possible arbitrary shifts between filters. The filter $H^h(z)$ is a CQF as the corresponding product filter $M(z)\equiv H^h(z)H^h(z^{-1})$ is halfband, i.e. $M(z)+M(-z)=1$. The product filter frequency response must be non-negative: $M(e^{j\omega})\ge 0$. The same statement applies for $H^g(z)$.

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Equation (2) can only be approximated with FIR filters. Therefore the Hilbert transform relationship (1) can only be approximated with FIR filters. The filters $H^h(z)$ and $H^g(z)$ will be referred to as the *real and imaginary CQF* respectively. For convenience it is assumed that the impulse response support of the CQF (must be of even length) is almost-centered-at-the-origin (ACO). An ACO filter has the support $n \in [-(L-1),L]$.

The class of Hilbert-pairs considered here has CQFs that are time-reversed versions of each other

$$H^g(e^{j\omega}) = e^{-j\omega} H^h(e^{-j\omega}) \tag{3}$$

The delay $e^{-j\omega}$ is needed to ensure that $H^g(e^{j\omega})$ is also ACO so that both filters are aligned. The CQFs are different spectral factors from the same product filter which is Symmetric-Self-Hilbertian (SSH) [12]:

Definition 1. A *SSH filter* is a product filter $M(z)$ with the following property: there exist at least two spectral factors, denoted by $H^h(z)$ and $H^g(z)$ that satisfy (3) with the corresponding complex wavelet spectrum satisfying

$$\Psi^A(\omega) \equiv \Psi^h(\omega) + j\Psi^g(\omega) \approx 0 \quad \text{for } \omega < 0$$

i.e. the complex wavelet $\psi^A(t) \equiv \psi^h(t) + j\psi^g(t)$ is approximately analytic.

Measures of approximation error can be defined as [12]

$$E_1 \equiv \frac{\max_{\omega<0}|\Psi^A(\omega)|}{\max_{\omega>0}|\Psi^A(\omega)|}, \quad E_2 \equiv \frac{\int_{-\infty}^{0} |\Psi^A(\omega)|^2 \, d\omega}{\int_{0}^{\infty} |\Psi^A(\omega)|^2 \, d\omega}$$

E_1 and E_2 measure the peak error and negative frequency energy respectively and the lower the values, the better the analytic quality. This definition of a SSH filter was first proposed in [12] and is also the one adopted in [13]. It does not specify the degree of approximation but is sufficient from a practical engineering perspective (and is sufficient for the discussions in this paper). To be more mathematically precise one can define a *SSH(ε) filter* to be a product filter satisfying the conditions above and having $E_1 \le \epsilon$ (or $E_2 \le \epsilon$). The parameter ε precisely specifies the degree of approximation.

The SSH filters in [12] yielded wavelets with almost maximum vanishing moments (VM). The basic idea in [12] was to introduce one degree of freedom in the product filter which is then optimized with respect to the analytic quality. Only the spectral factor with an approximately linear phase response was considered in [12]. Extensions that considered all spectral factors and allowing two degrees of freedom were proposed in [13] which yielded better analytic quality. However there is no control of the frequency response sharpness in [12,13] and all the filters in [12,13] have reduced sharpness compared to the maximum VMs filters of Daubechies. This paper extends the works in [12,13] to allow the design of sharper filters. The technique proposed here, called Extended-Zero-Pinning (EZP), is based on an extension of the Zero-Pinning (ZP) technique [14]. The wavelet pairs here are mirror image of each other, i.e. $\psi^g(t) = \psi^h(T-t)$ (where T is a constant), and the complex wavelet has the following symmetry $\psi^A(t) = (-j\psi^A(-t))^*$, i.e. the envelope $|\psi^A(t)|$ is mirror symmetric. There has also been other proposed design techniques for Hilbert-pairs but the wavelets do not have such symmetry. Furthermore in most previous works, for orthonormal pairs, all the design effort is usually focused on achieving the best approximation to (1). Using the EZP technique, it is relatively easy to explicitly include other criteria such as transition band sharpness and stopband ripple amplitude in the design. Reviews of earlier design techniques can be found in [7] and [15]. More recent techniques appear in [16–20]. In [20] the original ZP (without extension) is used to design minimum phase CQFs yielding highly non-symmetric wavelets. Most of the reported techniques deals with FIR filters with real coefficients but there has also been some techniques for (real coefficients) IIR filters [21,22] and complex coefficients FIR filters [23] (which require complex arithmetic for their implementation).

An outline of the paper is as follows. Section 2 presents the principle behind EZP after a brief review of the Parametric Bernstein Polynomial. The important issue of non-negativity is also discussed here. The optimization procedure for designing Hilbert-pairs using EZP is presented in Section 3. Two methods to speed up the optimization are presented here. Design examples are presented in Section 4 where relevant discussions are also found. The paper concludes with some comments in Section 5.

2. Extended-zero-pinning

The design of the product filter $M(z)$ is achieved through the use of the Parametric Bernstein Polynomial (PBP) which was introduced by Caglar and Akansu in [24]. The PBP can be written as

$$B(x) = K(x) - \sum_{l=0}^{(N-1)/2} k_l(x)\alpha_l \tag{4}$$

where

$$K(x) \equiv \sum_{i=0}^{(N-1)/2} \binom{N}{i} x^i (1-x)^{N-i}$$

and

$$k_l(x) \equiv \binom{N}{l} [x^l (1-x)^{N-l} - x^{N-l}(1-x)^l]$$

The product filter can obtained as $M(z) = B(-\frac{1}{4}z(1-z^{-1})^2)$ and it can be easily verified that $M(z)$ is halfband ($M(z)+M(-z)=1$), i.e. perfect reconstruction is structurally imposed. Furthermore it can also be shown [24] that if $\alpha_l = 0$ for $l = 0,\ldots,L$, then the CQF $H(z)$ has $(L+1)$ zeros at $z=-1$, i.e. the wavelet has $(L+1)$ VMs. The structural halfband and VM properties are the advantages of PBP in filter bank design. The set of non-zero Bernstein parameters: $\boldsymbol{\alpha}^{nz} \equiv [\alpha_{L+1},\ldots,\alpha_{(N-1)/2}]^T$ can be regarded as design parameters. The degrees of freedom available in the design process are

$$d_f \equiv \dim(\boldsymbol{\alpha}^{nz}) = (N-1)/2 - L.$$