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A graph cut strategy for transmission path problems in statistical energy analysis

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ABSTRACT

In this paper, it is shown how a strategy based on algorithms for computing cuts in undirected networks, can be applied to reduce energy transmission in realistic statistical energy analysis (SEA) models, with the sole modification of a limited number of internal and coupling loss factors. The frequency dependent case of SEA systems with multiple sources and targets is considered. A mathematical justification for the strategy is also provided, which relies on an analysis of the series expansion of the energy vector, in terms of the powers of the SEA graph adjacency matrix. A numerical example of vibroacoustic transmission in a simple test building has been included to show the performance of the approach.

1. Introduction

Let us consider an N-dimensional statistical energy analysis (SEA) model. Assume that external energy is being input at a limited number of say, N_s subsystems, whereas restrictions are posed on the energy another set of N_t subsystems should not surpass. Henceforth, let us term the former as *source* or *input* subsystems, while the latter subsystems, whose energy is to be reduced, will be referred to as the *target* or *output* subsystems. A common problem of noise and vibration control in SEA models is that of reducing the energy level at the N_t targets modifying as few SEA system parameters (internal and coupling loss factors) as possible. In other words, one aims at diminishing the vibration or noise levels at the target locations of the modelled physical system, carrying out a minimum number of design modifications.

To the best of our knowledge, the above control problem has been not addressed until recently, and a few options have been considered to solve it. If no information is available on which are the loss factors having more influence on the target subsystem energies, one can always resort to Monte Carlo and related approaches [1]. The values of a limited set of loss factors are randomly changed to new admissible values, and the subsystem energies become recomputed for each modification, until the desired energy goals at the targets are obtained. The use of genetic algorithms [2] can improve the final solution although they still not provide direct information on the explicit dependence of the target subsystem energies on the system parameters.

To remedy this situation and gain more insight on the SEA model behaviour, a more classical approach was followed in [3,4] that consists in performing a first-order Taylor expansion of the energies at the target subsystems, in terms of the

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system loss factors. Then, an optimization problem can be posed expressing the target subsystem energies as an objective function to be minimized by appropriate modification of the SEA system parameters. Obviously, the optimum solution would imply modifying a large number of loss factors, which could be infeasible for a real industrial case. Consequently, a sensitivity analysis has to be performed to assess the influence on the targets of the variations in the loss factors. As a result, modifications are only attempted for the most sensitive loss factors. A similar approach was followed in [5] for a SISO (single input/single output) SEA system, the main difference being in the definition of the cost function to be minimized. The latter was expressed as the summation of the contributions of several transmission paths [6,7] linking the source with the target. The method consequently depends on the number of paths taken into account, that rapidly increases when considering high order paths.

A third and totally different approach to the SEA vibroacoustic control problem is presented in this paper. Actually, the key idea of the method was already proposed in [8]. It was shown in that reference that several noise and vibration transmission path problems could be posed in the general framework of graph theory. The link between SEA and graph theory was carried out by means of the series expansion of the solution of the general SEA matrix system of equations [9]. It was observed that the generating matrix of this series could be viewed as a particular adjacency matrix of an equivalent SEA graph [8]. By assigning different weights to this matrix and combining path algebras [10] with standard linear algebra, it was shown how valuable information on the SEA system energy transmission paths could be obtained. More recently, it has been shown how algorithms intended to solve path problems in graphs could be adapted to efficiently compute and rank dominant paths in SEA systems [11]. On the other hand, one of the main topics addressed in [8] was the SISO control problem of trying to reduce the energy at a target subsystem that had been input to the SEA model through a single source subsystem. A strategy was developed that relies on the use of graph cut algorithms [12–14]. Given that all transmission paths from the source to the target have to cross a cutset [15,10,16] between them, it was proposed to find the cutset having minimum size and to modify the loss factors associated to it. If the desired energy reduction at the target was not achieved, cutsets of higher size were considered until the expected final energy level at the output was reached.

In this paper, we aim at pushing forward the graph cut strategy outlined in [8]. Whereas only low dimensional benchmark SISO graphs at a single frequency band were presented in [8], the first goal of this work will be that of extending the graph cut strategy to large MIMO (multiple input/multiple output) SEA graphs covering as much frequency bands as demanded. It will be shown that the MIMO case presents some important particularities. For instance, when cutting the graph, the subsets containing the sources and targets are not necessarily connected subgraphs, which allows to automatically recover the well-known noise control strategy of acting near the sources to reduce the emitted noise and vibrations. If this possibility is not good enough, the strategy automatically looks for alternatives, detecting structure bottlenecks where it may be worthwhile acting. The second goal of the present work is to provide a mathematical justification for the key step of the graph cut strategy. The justification relies on resorting the SEA system series solution in terms of the contributions of the paths between sources and receivers that cross the cutset, and then proceed to minimize the energy at the latter. Finally, the third goal of this paper is to show how the graph cut strategy could be applied in a realistic problem. The case of noise transmission between two rooms in a dwelling is addressed for this purpose.

When compared to other methods, the graph cut strategy could be helpful in solving some of their weakness, and actually it can be sought as an alternative but also as a complimentary method for them, in order to face SEA vibroacoustic control problems. For instance, a typical difficulty of the Monte Carlo method solutions is that they involve making design modifications on several disperse and disconnected locations of the structure, which are impractical in real life situations. In contrast, the subsystems to be modified are usually located in small areas for the graph cut approach, given that it automatically detects energy transmission bottlenecks [8]. This point is further reinforced by the fact shown in this work, that subsets from a SEA graph partition are not necessarily connected subgraphs, which allows for instance, to automatically check for solutions involving subsystems located near the sources and the targets. In what concerns optimization plus sensitivity analysis (see, e.g., [3,4] and references therein), the graph cut strategy could perform superior in those cases in which the internal loss factors of specially sensitive subsystems could not be drastically increased. In such situations, the SEA system energy will tend to redistribute and reach the target subsystems through other transmission paths, all of them being addressed in the graph cut case. However, the latter has the drawback that maybe a subsystem that will be worthwhile modifying is left apart if it does not belong to any computed cutset. Therefore, it is also worthwhile noting that the graph cut strategy could also be implemented in combination with optimization plus sensitivity analyses, or with Monte Carlo approaches. In the former case, this will ensure that no subsystem upon which it would be worthwhile to act is ignored. Concerning the latter, one could compute a small size graph cut and make a Monte Carlo computation on a reduced sample space. This will ease the computational cost and yield a less disperse solution. However, these combined possibilities will be not addressed in this work and are left for future developments.

The paper is organized as follows. In Section 2, a brief review of some basic SEA and graph theory topics that will be needed for the remaining of this work is given for completeness. In particular, emphasis is put on the series solution of the SEA matrix system, on the notion of SEA graph and on some connectivity issues belonging to graph theory. In Section 3, the general MIMO strategy for energy reduction at the target subsystems is first explained. Then a benchmark example is presented to highlight some features of the method and a mathematical justification is provided to support its key step. A numerical example to show how the graph cut strategy could operate in a realistic situation is given in Section 4. The problem of vibroacoustic transmission between separated rooms in the simple SEA model of a building is addressed. Final considerations conclude the paper in Section 5.