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A second-moment approach for direct probabilistic model updating in structural dynamics

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ABSTRACT

Discrepancies between experimentally measured data and computational predictions are unavoidable for structural dynamic systems. Model updating methods have been developed over the past three decades to reduce this gap. Well established model updating methods exist when both the model and experimental measurements are deterministic in nature. However in reality, experimental results may contain uncertainty, for example arising due to unknown experimental errors, or variability in nominally identical structures. Over the past two decades probabilistic approaches have been developed to incorporate uncertainties in computational models. In this paper, the natural frequencies and the eigenvectors of the system are measured and assumed to be uncertain. A random matrix approach is proposed and closed-form expressions are derived for the mean matrix and the covariance matrix of the updated stiffness matrix. A perturbation technique is used to obtain a usable expression for the covariance matrix.

The new method is illustrated by three numerical examples highlighting the influence of the eigenfrequency uncertainties on the mean matrix and the influence of the eigenvector uncertainties on the covariance matrix.

1. Introduction

In various areas of computational modeling, it has been established over the past three decades that uncertainties should be taken into account for credible predictions. In the context of structural dynamics, such uncertainties can be broadly divided into two categories, namely, parametric uncertainty and non-parametric uncertainty. Parametric uncertainty includes uncertainty in geometric parameters, friction coefficient, and the moduli of the materials involved. In contrast, non-parametric uncertainty can arise due to the lack of scientific knowledge about the model which is unknown a priori. Although this distinction is often made, the origin of uncertainty in real-life systems is not always obvious [1,2]. In this paper we propose an analytical approach to update random variables describing parametric uncertainty of a linear dynamical system using experimental data.

The deterministic finite element model updating problem [3–7] is well established, both in the development of methods and in application to industrial-scale structures [8,9]. The proposed methods can be broadly divided into two categories, namely, the non-parametric (or direct) and parametric approaches. In the non-parametric approach, developed

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Nomenclature

$\mathrm{cov}(\Omega^2)$	covariance matrix of Ω^2
$\|X\|_S$	stochastic norm of X
$\tilde{Y}$	mean of Y estimated through a perturbation method
$C=\mathrm{cov}(X)$	covariance matrix of X
$(\Sigma;\Psi)$	Kronecker product decomposition of a covariance matrix
Ω	natural frequency matrix
$\otimes$	Kronecker product of two matrices
$\overline{X}$	mean of X
$\|X\|_F$	Frobenius norm of X
$\mathrm{tr}(X)$	trace of X
$\mathrm{vec}(\cdot)$	vectorization operator of a matrix
$\widehat{X}$	maximum likelihood estimator of X
$E\{\bullet\}$	expectation operator
$f(t)$	forcing vector
K	initial estimate of the stiffness matrix
M	mass matrix
n	number of degrees of freedom
$q(t)$	response vector
X	mode shape matrix
X'	transpose of X
Y	corrected stiffness matrix

during early eighties, the system matrices (mass, stiffness and damping matrices) are updated directly so that the differences between the predicted data (natural frequencies, damping ratios, and mode shapes) and measured data are minimum according to a suitable norm. The non-parametric methods have some significant problems that have restricted their application in practical examples where the model has a large number of degrees of freedom. The methods do not give a clear physical insight into the modeling errors that are corrected, and the connectively of the original model is not necessarily preserved. Furthermore the incompleteness of the measured data, in terms of the number of sensors used and the number of modes measured, leads to a highly underdetermined estimation problem. This is further complicated because the lower frequency modes are measured but the higher frequency modes have the greatest effect on the stiffness matrix. Despite these problems non-parametric methods can be applied to reduced order models for some limited objectives, such as controller performance or response prediction studies. The alternative, and increasingly popular, approach is parametric model updating where physical parameters (for example, joint stiffnesses, thicknesses) are selected and updated. The estimation is usually based on some kind of sensitivity analysis that minimizes the error between predicted results and test data from a single physical structure. The choice of updating parameters is an important aspect of the process and should always be justified physically. Model uncertainties should be located and parameterized sensitively to the predictions. Finally, the model should be validated by assessing the model quality within its range of operation and its robustness to modifications in the loading configuration, design changes, coupled structure analysis and different boundary conditions.

Collins et al. [10] developed a Bayesian approach to model updating using linearized sensitivities based on knowledge of the distributions of the unknown parameters and the vibration measurements. In these approaches, the randomness arises only from the measurement noise and the updating parameters take unique values, to be found by iterative correction to the estimated means, whilst the variances are minimized [11]. These statistical approaches have been extended to update parameter distributions using measured response distributions from multiple measurements. These include Bayesian methods [12–15], perturbation based methods [16] and the maximum likelihood method [17]. Hua et al. [18] considered an improved perturbation method where statistical correlations between the updating parameters were taken into account. Faverjon et al. [19] considered updating of uncertain systems using a polynomial chaos expansion. McFarland et al. [20] used Gaussian process emulators to update linear systems with parametric updating. Their approach is valid for both Gaussian and non-Gaussian random variables. Ren et al. [21] proposed a response surface-based finite-element-model updating technique using structural static responses. Adhikari and Friswell [22] proposed a sensitivity based model updating approach for distributed updating parameters expressed using the Karhunen–Loève expansion. Goller et al. [23] considered stochastic model updating for complex aerospace structures. More recently Khodaparast et al. [24] considered model updating with an interval description of uncertain variables.

The source of the uncertainty in the model must be considered when the objectives of the model updating problem are considered. The uncertainty may be epistemic, for example arising from the measurement process, that may be reduced by using more information. Alternatively the uncertainty may be aleatory, for example arising from manufacturing or material variability for multiple structures, that cannot be improved by increased information on a single structure. Determining the source of the uncertainty in a practical problem is difficult, and many updating methods do not make a clear distinction. Indeed the way a method is used can determine the type of uncertainty considered. For example, the Bayesian methods [10,11] specify the covariance matrix of the measurements, and the original papers assumed this represented the measurement errors with a view to estimating deterministic parameters for a single structure. However if the covariance matrix represented the variability in measurements taken on multiple structures then the methods could be used to estimate the covariance of the estimated parameters.

The majority of the research reported in the literature consider the parametric approach for stochastic model updating. The aim of this paper is to propose a simple and computationally efficient approach (i.e., avoiding Monte Carlo Simulation) to update stiffness matrices from experimental measurements of the natural frequencies and the corresponding eigenvectors. A direct second moment approach based on random matrix theory is developed in this paper. The outline of the paper is as follows. In Section 2 the direct deterministic updating methods are briefly presented. Then in Section 3, the context of the