Mechanical Systems and Signal Processing 29 (2012) 474–484

Contents lists available at SciVerse ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Permutation entropy: A nonlinear statistical measure for status characterization of rotary machines

Ruqiang Yan [a,*], Yongbin Liu [b], Robert X. Gao [c]

[a] School of Instrument Science and Engineering, Southeast University, Nanjing, Jiangsu 210096, P.R. China
[b] School of Electrical Engineering and Automation, Anhui University, Hefei, Anhui 230039, P.R. China
[c] Department of Mechanical Engineering, University of Connecticut, Storrs, CT 06269, USA

ARTICLE INFO

Article history:
Received 12 April 2011
Received in revised form
29 September 2011
Accepted 25 November 2011
Available online 16 December 2011

Keywords:
Permutation entropy
Nonlinear dynamics
Fault diagnosis

ABSTRACT

This paper investigates the usage of permutation entropy for working status characterization of rotary machines. As a statistical measure, the permutation entropy describes complexity of a time series or signal measured on a physical system through phase space reconstruction, and takes into account non-linear behavior of the time series, as often seen in vibration signals of rotary machines. Thus it can be served as a viable tool for detecting dynamic changes of the machine working status. The effect of embedded dimension and time delay on calculation of the permutation entropy value has been studied, and the validity of the permutation entropy for detecting dynamic change of a physical system is studied through a well known non-linear system, the Logistic Map. Comparison with other complexity measures using a numerically formulated signal has also been investigated. Experimental results on bearing vibration analysis have then confirmed that the permutation entropy provides an effective measure for monitoring the working status of rolling bearings.

1. Introduction

To evaluate the working status of rotary machines, vibration signals generated from these machines have often been measured and then analyzed in detail [1–3]. As fault information of rotary machines is mostly reflected by singular points of abrupt changing signals, detecting dynamic change of the vibration signals in time is important for early fault identification [4]. During the process of machine operations, some phenomena, such as strike, velocity chopping, structure transmutation, and friction, may occur. These phenomena cause the vibration signals generated from the machines to be non-stationary and nonlinear. As a result, traditional linear methods may not effectively detect dynamic change of those vibration signals. With the development of nonlinear dynamic theory, a number of methods have been presented to detect such changes. These include nonlinear cross prediction analysis [5], lyapunov exponent [6], correlation dimensions [7], and symbolic dynamics [8]. Most of these methods are based on quantifying certain aspects of the nearest neighbors in phase space, and, as a result, are computationally expensive.

The Complexity Measure, in comparison, is computationally more efficient [9]. The complexity of a signal can be described by the Lempel–Ziv Complexity (LZC) [10,11], Approximate Entropy (ApEn) [12,13], and Permutation Entropy (PE) [14]. The Lempel–Ziv Complexity indicates the degree of regularity of a time series in one dimension, and calculation of complexity values on vibration signals measured from a large rotating machine [15] has shown that the inception and growth of faults in the machine could be correlated with changes in the LZC value. Another study on defects detection in

* Corresponding author. Tel.: +86 135 84054760; fax: +86 25 83794158.
E-mail addresses: ruqiang@seu.edu.cn (R. Yan), lyb@ustc.edu.cn (Y. Liu), rgao@engr.uconn.edu (R.X. Gao).

0888-3270/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ymssp.2011.11.022

rolling bearings has established quantitative relationship between the LZC value and the defect size, and consequently the defect severity level [16]. Approximate Entropy, on the other hand, expresses the regularity of a time series in multiple dimensions, and contains more time-related information [13]. In a study on turbo generator, it was found that the ApEn values have increased significantly after looseness of the bearing bushing was identified, when compared with normal operation conditions [17].

Different from Lempel–Ziv Complexity and Approximate Entropy, Bandt and Pompe presented Permutation Entropy, a parameter of average entropy, to describe the complexity of a time series [14]. Because the permutation entropy makes only use of the order of the values, it is robust under non-linear distortion of the signal, and is also computationally efficient [18]. It has been successfully applied to a number of applications. For example, permutation entropy has shown to be effective in detecting vigilance changes and preictal states from scalp EEG [19] and tracking transient dynamics of EEG recordings [20]. It has also provided an advantageous complexity estimation to improve effectiveness on fetal behavior states classification [21]. For financial time series analysis, the permutation entropy has been used together with symbolic dynamics to perform a non-parametric independent test [22]. In the field of manufacturing, permutation entropy has been used for online chatter detection in turning process [23], and tool flute breakage detection in end milling [24]. It has been verified that compared with other parameters, such as Lyapunov exponent and fractal dimensions, permutation entropy can detect dynamic change in complex systems more effectively [25,26] . Furthermore, an overview on both theory and applications of permutation entropy is introduced in [27] in detail, and with more and more researchers have devoted to this topic, more theoretical insights on permutation entropy can be seen in [28–30].

Motivated by these prior efforts, this paper investigates the utility of the Permutation Entropy for detecting dynamic change in vibration signals of rotary machine/machine components, with specific application in rolling bearings. After introducing the theoretical background of the permutation entropy, its validity for detecting dynamic changes, the effect of embedded dimension and time delay on the calculation of PE, and computational cost, are studied. Comparison with other complexity measures are then conducted using a numerically formulated signal. After that, the effectivenss of the permutation entropy on characterizing working status of rolling bearings is experimentally verified.

2. Theoretical framework of permutation entropy

The mathematical theorem of the permutation entropy was described in detail in [14,18,31]. According to the Takens–Maine theorem, the phase space of a time series $\{x(i),\ i=1,2,\ldots,N\}$ can be reconstructed as

$$\begin{cases} X(1)=\{x(1),x(1+\tau),\ldots,x(1+(m-1)\tau)\} \\ \vdots \\ X(i)=\{x(i),x(i+\tau),\ldots,x(i+(m-1)\tau\} \\ \vdots \\ X(N-(m-1)\tau)=\{x(N-(m-1)\tau),x(N-(m-2)\tau),\ldots,x(N)\} \end{cases} \tag{1}$$

where m is the embedded dimension and τ is the time delay. As described in [31] by Cao et al., the m number of real values contained in each $X(i)$ can be arranged in an increasing order as

$$\{x(i+(j_1-1)\tau)\le x(i+(j_2-1)\tau)\le\ldots\le x(i+(j_m-1)\tau)\} \tag{2}$$

If there exist two or more elements in $X(i)$ that have the same value, e.g. $x(i+(j_1-1)\tau)=x(i+(j_2-1)\tau)$, their original positions can be sorted such that for $j_1\le j_2$, $x(i+(j_1-1)\tau)\le x(i+(j_2-1)\tau)$ can be written. Accordingly, any vector $X(i)$ can be mapped onto a group of symbols as [31]

$$S(l)=(j_1,j_2,\ldots,j_m) \tag{3}$$

where $l=1,2,\ldots,k$ and $k\le m!$ ($m!$ is the largest number of distinct symbols). $S(l)$ is one of the $m!$ symbol permutations, which is mapped onto the m number symbols $(j_1,j_2,\ldots,j_m)$ in m-dimensional embedding space. If $P_1,P_2,\ldots,P_k$, are used to denote the probability distribution of each symbol sequences, respectively, and $\sum_{l=1}^{k} P_l=1$, then the permutation entropy of order m for the time series $\{x(i),\ i=1,2,\ldots,N\}$ can be defined as the Shannon entropy for the k symbol sequences as

$$H_P(m)=-\sum_{l}^{k} P_l \ln P_l \tag{4}$$

The maximum value of $H_p(m)$ can be obtained as $\ln(m!)$ when all the symbol sequences have the same probability distribution as $P_l=1/m!$. Therefore, the permutation entropy of order m can be normalized as [31]

$$0\le H_P=H_P/\ln(m!)\le 1 \tag{5}$$

For simplicity, H_p is used to replace $H_p(m)$ in the following sections. The value of H_P can represent the randomicity of the time series $\{x(i),i=1,2,\ldots,N\}$, and it describes local order structure of the time series. The smallest possible value of H_p is zero, which means that the time series is very regular [14]. The largest possible value of H_p is 1, which is realized when all permutations have equal probability, as is in the case of white noise. The smaller the value of H_p, the more regular the time series is. The change of H_p can reflect and magnify the subtile transformation of the time series [14].