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A panorama on multiscale geometric representations, intertwining spatial, directional and frequency selectivity

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ABSTRACT

The richness of natural images makes the quest for optimal representations in image processing and computer vision challenging. The latter observation has not prevented the design of image representations, which trade off between efficiency and complexity, while achieving accurate rendering of smooth regions as well as reproducing faithful contours and textures. The most recent ones, proposed in the past decade, share a hybrid heritage highlighting the multiscale and oriented nature of edges and patterns in images. This paper presents a panorama of the aforementioned literature on decompositions in multiscale, multi-orientation bases or dictionaries. They typically exhibit redundancy to improve sparsity in the transformed domain and sometimes its invariance with respect to simple geometric deformations (translation, rotation). Oriented multiscale dictionaries extend traditional wavelet processing and may offer rotation invariance. Highly redundant dictionaries require specific algorithms to simplify the search for an efficient (sparse) representation. We also discuss the extension of multiscale geometric decompositions to non-Euclidean domains such as the sphere or arbitrary meshed surfaces. The etymology of panorama suggests an overview, based on a choice of partially overlapping "pictures". We hope that this paper will contribute to the appreciation and apprehension of a stream of current research directions in image understanding.

1. Introduction: vision aspects, scope and notations

1.1. Background on vision aspects of scale

Many natural-world object features are substantive only over a certain spatial extent. In other words, the scale of observation is crucial in object recognition and understanding. For instance, a chair would be easily recognizable in the scale of a few meters. But neither at a centimeter scale which captures the chair's texture and not its object appearance, or at a hectometer scale, where the chair's appearance is hardly distinguished from other surrounding objects.

Accordingly, early neurophysiological studies in biologic perception reveal that those objects are generally apprehended differently according to the scale of observation by the sensory receptors and the cortex of mammalians [1,2]. Efficient information extraction is thus required for artificial sensing systems to mimic standard biologic tasks such as object recognition.

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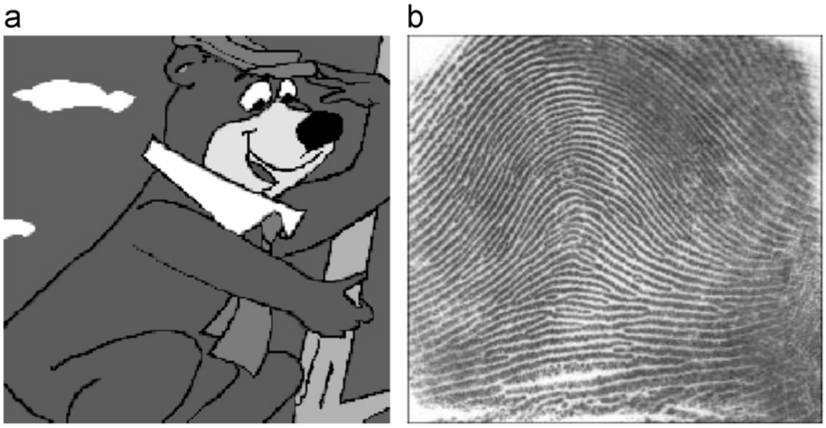

Fig. 1. Two faces of the cartoon-texture model: (a) Yogi bear and (b) Fingerprint.

Pixel-based representations as linear combinations of "delta" functions suffice for simple data manipulation but are very limited for higher level tasks. Only assuming some sufficient resolution in the data, the lack of prior knowledge in the extent of objects to be analyzed calls for tools able to unveil the appropriate scales and to allow a hierarchical representation of the underlying features [3–5]. Disregarding the peculiar fractal formalism [6,7] where similar phenomena appear at different scales (what is called *self-similarity*), special attention has been paid to data transformations able to capture object features over a range of scales in a more compact form. Sparsity, amounting to a reduced number of parameters in a suitable domain, is thus used as a heuristic guide to image understanding. Bearing analogies with findings in vision processes [8], several sparse decompositions have proven efficient in image compression, with the discrete wavelet transform (DWT) as their most well-known avatar, often intermingled with information theory and technical wizardry, from bit plane arithmetic coding [9] to trellis coded quantization. A compact history and a paper collection are given in [10,11], respectively.

Yet, beyond image compression transforms, other decomposition techniques are needed, with more resolving power in complex scene detection, denoising, segmentation or, in a broad sense, scene understanding. As a matter of fact, standard separable wavelet transforms appropriately detect point-like (0-D) singularities and address mild noise levels. Still, they generally lack performance in dealing with higher dimensional features combining both regularity and singularity such as edges, contours or regular textures, that may also be anisotropic. Amongst their limitations are shift sensitivity, limited orientation selectivity, rigid and uneven atom shapes (*e.g.*, fractal-looking asymmetric Daubechies wavelets), crude frequency direction selection. Major challenges reside in a proper definition of the underlying regularity (with respect to each feature) and corresponding singularities. These challenges are amplified by additional degradations from which acquired data may suffer such as blur, jitter and noise. Descriptive mathematical models of images combining cartoon and textures become increasingly popular [12,13] and progressively yield tractable algorithms. We note that there exists a continuum of real-world images between cartoon and textures, ranging from cartoon-ish Yogi bear in Fig. 1(a) to "textural" fingerprints in Fig. 1(b). In between these two extreme image types, there exists many possible variations in image object complexity.

Moreover, both contours and (even regular) textures are known to be ill-defined. They are indeed viewer- and scale-dependent concepts in discrete images or volumes. Consider an image resulting from a combination of piecewise smooth components, contours, geometrical textures and noise. Their discrimination is required for high level image processing tasks. Each of these four components could be detected, described and modeled by different formalisms: smooth curves or polynomials, oriented regularized derivatives, discrete geometry, parametric curve detectors (such as the Hough transform), mathematical morphology, local frequency estimators, optical flow approaches, smoothed random models, etc. They have progressively influenced the hybridization of standard multiscale transforms towards more geometric and sparser representations of such components, with improved localization, orientation sensitivity, frequency selectivity or noise robustness.

1.2. *Scope of the paper*

Geometry driven "$\star$−let" transforms [14] have been popular in the past decade, with a seminal ancestor in [15]. Early [16], a debate opened on the relative strength of Eulerian (non-adaptive) versus Lagrangian (adaptive) representation, now pursued with the growing interest in dictionary learning [17].

As of today, the authors believe that the discussion is not fully settled in the various different uses of sparsity in images. Neither has the trade-off between redundancy and sparsity. A number of early papers on geometric multiscale methods appear in [18]. Comparisons are drawn in [19,20], while [21–24] focus on ridgelets, curvelets and wedgelets, as representative of fixed and adaptive decompositions. The present paper aims at providing a broader panorama of the recent developments in multiscale decompositions targeted