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Multitaper Wigner and Choi–Williams distributions with predetermined Doppler–lag bandwidth and sidelobe suppression ☆

Maria Hansson-Sandsten *

Mathematical Statistics, Centre for Mathematical Sciences, Lund University, Box 118, SE–22100 Lund, Sweden

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ABSTRACT

In this paper, a penalty function is designed and used in the computation of multitapers which correspond to the Wigner and Choi–Williams distributions. The resulting multitaper spectrogram will approximately fulfill the concentration of the distribution but will additionally suppress the cross-terms outside a predetermined Doppler–lag bandwidth. The specific region as well as the amount of cross-term suppression is determined by parameters of the penalty function. The proposed method uses a limited number of multitapers which results in computationally efficient calculations. The time–frequency concentration and resolution of the proposed method and the original distribution are compared and the performance for signals disturbed by white noise is also evaluated. Estimation of event related potentials disturbed by EEG exemplify the use of the method.

1. Introduction

In the area of time–frequency analysis, a large number of time–frequency distributions have been proposed for different applications. From time–frequency concentration viewpoint, the Wigner distribution is the optimal choice. However, the so-called cross-terms disturbing the auto-terms from multi-component signals are a major drawback. Today, a number of other time–frequency distributions exist with different ability to suppress the resulting cross-terms from the Wigner distribution, e.g., [1–4].

A computationally efficient algorithm that corresponds to a time–frequency distribution can be found using a multitaper spectrogram, especially if the number of averaged spectrograms can be small and also by using symmetry of matrices [5]. This motivates the implementation of time–frequency kernels as multitaper spectrograms. The corresponding eigenvalues and eigenvectors of the rotated time-lag kernel of a specific distribution are used as weights and tapers in the multitaper spectrogram [6]. The phrase multitaper was originally introduced by Thomson, for the case of stationary processes with smooth spectra [7]. One of the advantages of the Thomson multitapers is strong sidelobe suppression outside a predetermined frequency interval. Other methods have been proposed for the multitaper spectrum estimate of stationary processes where the tapers also fulfil the criterion of strong sidelobe suppression [8,9]. Multitaper decompositions have been analyzed from several aspects, for existing distributions, e.g., [10,11], and new multitaper techniques for non-stationary signal analysis have also been proposed, e.g., [12–15]. The aspect of time–frequency localization and orthogonality in the time–frequency domain (in contrast to only considering the frequency domain) was noted by [16] and made the Hermite functions to become often used as multitapers for spectrogram estimation of non-stationary processes [14,17–19].

In some contributions, the weighting of the different multitaper spectrograms is optimized for fixed tapers

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* Tel.: +46 46 222 49 53.
E-mail address: sandsten@maths.lth.se

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(usually the Hermite functions) [14,20]. In these cases a model spectrum of data is needed for the optimization. In many practical cases, the spectrum of the signal to be estimated is unknown, but more vague information could be available, e.g., approximate concentration and resolution of time–frequency components.

Another important case is signals disturbed by additive noise, where it is notable that orthogonal multitapers are optimal from a (white) noise variance reduction aspect. The theoretical results for computing the bias and variance of the Wigner distribution for the case of additive noise are given in [21] and a minimum-variance kernel is obtained in [22].

Therefore, in this paper, the aim is to find the multitapers corresponding to the Wigner and Choi–Williams distributions, as these are often utilized and are known to have an appropriate concentration, but often give cross-terms between different time–frequency components. By incorporating a suppression level of disturbances outside a predefined Doppler–lag bandwidth, the concentration is retained but the cross-terms are suppressed. A penalty function is proposed and a generalized eigenvalue problem is solved to find the multitapers and weights for the spectrogram calculation. One of the main advantages of the approach is that only some general ideas of concentration and resolution of components are needed. Using this information together with a pre-defined suppression level of cross-terms define the penalty function. The resulting multitaper spectrogram includes just a few averages and the computed tapers have the orthogonality property that guarantee uncorrelated multitaper spectrograms, which affects the reduction of white noise disturbance of the estimate.

An initial idea was presented in [23] using another penalty function and limiting the Thomson multitaper kernel. In Section 2 the spectrogram decomposition of time–frequency distributions is described. The penalty matrix that is used to suppress the cross-terms are presented in Section 3 followed by examples of multitapers and weights. An evaluation of the proposed methods are given in Section 4, and in Section 5 examples of estimation of event-related potentials from the brain are shown. Section 6 concludes the paper.

2. Spectrogram decomposition of time–frequency distributions

The connection between a multitaper spectrogram and a smoothed Wigner distribution is found using the following approach. The multitaper spectrogram is defined as

$$S_x(t,f) = \sum_{k=1}^{K} \alpha_k \left| \int_{-\infty}^{\infty} h_k^*(t-t_1)x(t_1)e^{-i2\pi f t_1}\, dt_1 \right|^2$$
$$= \sum_{k=1}^{K} \alpha_k \left(\int_{-\infty}^{\infty} h_k^*(t-t_1)x(t_1)e^{-i2\pi f t_1}\, dt_1 \right) \times \left(\int_{-\infty}^{\infty} h_k(t-t_2)x^*(t_2)e^{i2\pi f t_2}\, dt_2 \right), \tag{1}$$

where $x(t)$ is the signal, α_k, $k=1\ldots K$, are the weights and $h_k(t)$, $k=1\ldots K$, are the multitaper functions. With $t_1 = t'+\tau/2$ and $t_2 = t'-\tau/2$,

$$S_x(t,f) = \sum_{k=1}^{K} \alpha_k \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\left(t'+\frac{\tau}{2}\right)x^*\left(t'-\frac{\tau}{2}\right) \times h_k\left(t-t'-\frac{\tau}{2}\right)h_k^*\left(t-t'+\frac{\tau}{2}\right)e^{-i2\pi f\tau}\, d\tau\, dt'. \tag{2}$$

We identify the instantaneous autocorrelation function as

$$r_x(t,\tau) = x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right) \tag{3}$$

and the time-lag kernel as

$$\rho(t,\tau) = \sum_{k=1}^{K} \alpha_k h_k\left(t+\frac{\tau}{2}\right)h_k^*\left(t-\frac{\tau}{2}\right) \tag{4}$$

giving the quadratic class of time–frequency distributions, e.g., [24, Chapter 3], as

$$C_x(t,f) = S_x(t,f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_x(t',\tau)\rho(t-t',\tau)^* e^{-i2\pi f\tau}\, dt'\, d\tau. \tag{5}$$

Defining

$$\rho^{rot}(t_1,t_2) = \rho\left(\frac{t_1+t_2}{2}, t_1-t_2\right) \tag{6}$$

and if the kernel $\rho^{rot}(t_1,t_2)$ satisfies the Hermitian property

$$\rho^{rot}(t_1,t_2) = (\rho^{rot}(t_2,t_1))^*$$

then solving the integral

$$\int \rho^{rot}(t_1,t_2)q(t_1)\, dt_1 = \lambda q(t_2) \tag{7}$$

results in eigenvalues λ_k and eigenfunctions $q_k(t)$, which form a complete set that can be used as weights, α_k, and multitaper functions, $h_k(t) = q_k(t)$, $k=1\ldots K$, in Eq. (1).

3. Penalty matrices and multitapers

In [9], a frequency penalty function was used to suppress the sidelobes of the mean squared error optimal multitapers for the case of stationary processes with peaked spectrum. A similar idea is proposed here with a penalty function

$$W_{v\tau}(t_1,t_2) = \begin{cases} P\cdot r_P(t_1-t_2) & \text{if } |t_1| \geq \frac{\Delta\tau_p}{2} \text{ and } |t_2| \geq \frac{\Delta\tau_p}{2}, \\ r_P(t_1-t_2) & \text{otherwise.} \end{cases} \tag{8}$$

The covariance penalty function is defined as

$$r_P(\tau) = P\cdot\delta(\tau)-(P-1)\cdot\Delta v_p \operatorname{sinc}(\Delta v_p \tau), \tag{9}$$

with $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$, where the corresponding spectral density penalty function is

$$S_P(f) = \begin{cases} P & \text{if } |f| \geq \frac{\Delta v_p}{2}, \\ 1 & \text{if } |f| < \frac{\Delta v_p}{2}. \end{cases} \tag{10}$$

The definition of the spectral density penalty function is similar to the one in the stationary case in [9]. This frequency penalty function, that suppress a factor of P outside a predetermined interval Δv_p, is combined with the block structure that suppress outside the interval $\Delta\tau_p$.