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Probabilistic uncertainty analysis of an FRF of a structure using a Gaussian process emulator

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ABSTRACT

This paper introduces methods for probabilistic uncertainty analysis of a frequency response function (FRF) of a structure obtained via a finite element (FE) model. The methods are applicable to computationally expensive FE models, making use of a Bayesian metamodel known as an *emulator*. The emulator produces fast predictions of the FE model output, but also accounts for the additional uncertainty induced by only having a limited number of model evaluations. Two approaches to the probabilistic uncertainty analysis of FRFs are developed. The first considers the uncertainty in the response at discrete frequencies, giving pointwise uncertainty intervals. The second considers the uncertainty in an entire FRF across a frequency range, giving an uncertainty envelope function. The methods are demonstrated and compared to alternative approaches in a practical case study.

1. Introduction

Finite element (FE) modelling is perhaps the most widely used computational tool in the analysis of structural vibrations, particularly for the prediction of frequency response functions (FRFs). In recent years there has been a growing level of interest in how different types of uncertainty can be handled with this modelling approach. These uncertainties can be inherent to the model itself (for example due to assumptions regarding the boundary conditions), or alternatively they could arise due to unknown values of physical parameters (for example component geometry or material properties). In the latter case, this lack of knowledge could be attributed to variation between nominally identical components (i.e. variability), or uncertainty during the design process regarding the final choice of dimensions or material.

There has long been interest in how uncertainty propagates through FE models. The method with the greatest pedigree is the stochastic finite element (SFE) method [1]; this is a probabilistic method. In the general SFE formulation, the material properties across the structure can be specified as a random field. In a manner similar to the discretisation of the structure into finite elements, the random field is discretised into a denumerable set of random variables using the Karhunen–Loeve expansion, which is then truncated at some finite order. The results from the FE model are then expressed as a mean value supplemented by an expansion in terms of the random variables, allowing statistics of the quantity of interest to be computed. In the last decade, interest has grown in possibilistic approaches, such as a fuzzy approach to FE analysis and computation of modal quantities [2,3]. More recent work has considered component mode synthesis as a framework for

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investigating both probabilistic and possibilistic uncertainties [4], and using possibilistic techniques based upon fuzzy numbers [5]. A 'fuzzy FE' approach is also developed in [6] and applied to a variety of case studies including the Garteur benchmark FE problem—a small scale aircraft model developed for assessing ground vibration test techniques.

A common issue when trying to propagate uncertain parameters through complex FE models is that the deterministic nature of the modelling approach leads to many model evaluations being performed, each for a different configuration of the uncertain inputs. In probabilistic modelling, this results in Monte Carlo simulations, whilst in possibilistic modelling, the repeated model evaluations can be used to generate fuzzy numbers representing the uncertainty in the model's response.

This paper focusses on a probabilistic method for uncertainty analysis of FE models, using a statistical metamodel, or emulator, to reduce the number of FE model evaluations required, and hence reduce the computational cost. The remainder of the paper is organised as follows. First, the probabilistic uncertainty analysis problem is formulated, before introducing the concept of the emulator. Next, the use of the emulator for uncertainty analysis is described using a simple graphical example. The uncertainty analysis of FRFs that are predicted from FE models is then considered. This approach is then applied to a numerical case study based upon the Garteur testbed. Following a discussion, conclusions are drawn regarding the application of this modelling approach to FE modelling problems in structural dynamics.

2. Probabilistic uncertainty analysis of FE models

Consider a deterministic FE model evaluated at a particular degree of freedom. It takes a set of p input parameters, denoted as $\mathbf{x}=(x_1,\ldots,x_p)^T$, and returns a set of outputs that consist of pairs of modal parameters. A typical FE analysis considers a subset of the modal parameters, which we denote $\{(\hat{m}_i,\hat{k}_i): i=1,\ldots n_{modes}\}$, where $\hat{m}_i$ are the modal masses and $\hat{k}_i$ are the modal stiffnesses. The FE model is deterministic, so repeated runs with the same configuration of input parameters will return the same outputs, and we may represent it as a function $\mathbf{y}=\boldsymbol{\eta}(\mathbf{x})$, where $\mathbf{y}=(\hat{m}_1,\hat{k}_1,\hat{m}_2,\hat{k}_2,\ldots,\hat{m}_{n_{modes}},\hat{k}_{n_{modes}})^T$.

In the probabilistic uncertainty analysis of a deterministic computer model, we consider the values of the uncertain input parameters to be a multivariate random variable X. As a result, the output of the model is also a multivariate random variable, which we denote $Y=\boldsymbol{\eta}(X)$. The first step in the analysis is to quantify the uncertainty in X by specifying a probability distribution $F(\mathbf{x})$. This distribution may be constructed using data, or by eliciting expert opinion [7], or a combination of both. Our aim is then to propagate the uncertainty in X through the computer model in order to characterise the distribution of Y, which is known as the *uncertainty distribution*.

A straightforward solution to this problem is to use a Monte Carlo procedure. In this we draw a large sample $\{\mathbf{x}_1,\ldots,\mathbf{x}_N\}$ from the input distribution $F(\mathbf{x})$ and run the model at each sampled input configuration $\mathbf{x}_i$. The result is a sample of the outputs $\{\mathbf{y}_1,\ldots,\mathbf{y}_N\}$, from which we can estimate any summary of the uncertainty distribution such as the mean, the variance, or a particular quantile, using the corresponding summary statistic. For example, the mean of the uncertainty distribution may be estimated using the sample mean of $\{\mathbf{y}_1,\ldots,\mathbf{y}_N\}$. For a general summary, denoted $S(Y)$, the precision of the estimate is determined by the sample size N, and standard techniques are available for estimating the Monte Carlo error in the estimate [8].

When we perform an uncertainty analysis of an FE model of a structure, characterising the uncertainty distribution of the FE model outputs (i.e. the modal parameters) is often only an interim step. In many cases, we are ultimately interested in quantifying the uncertainty in the corresponding FRF of the modelled structure. According to the concept of modal superposition, the FRF of the undamped structure is calculated as

$$G(\omega;\mathbf{y})=\sum_{i=1}^{n_{modes}}\frac{1}{\hat{k}_i-\omega^2\hat{m}_i}. \tag{1}$$

Since the modal parameters are uncertain, the FRF at a particular frequency ω is itself a random variable, which we denote G_ω. Given the Monte Carlo sample of the modal parameters, we may obtain a sample from the distribution of the FRF at ω by simply plugging the sampled modal parameters into Eq. (1). This gives us a sample $\{G(\omega;\mathbf{y}_1),\ldots,G(\omega;\mathbf{y}_N)\}$ from which we may obtain any summary of the FRF uncertainty distribution, $S(G_\omega)$.

In the next section, an alternative approach is described based upon the use of an emulator. However, at this stage it is useful to briefly mention the practical relevance of Eq. (1). Real structures possess damping, such that the modal solution involves an imaginary term. Nevertheless, most FE solutions do not consider structural damping, and so any uncertainty in the damping does not directly influence the problem of uncertainty propagation in the FE analysis. Another aspect of Eq. (1) is that there are more generalised modal solutions that involve mass-normalised modes and modal constants, rather than mass and stiffness terms. Furthermore, uncertainty can cause the density functions for the natural frequencies to overlap and give a finite probability that mode i will appear at a higher frequency than mode $i+1$. This means that the emulator cannot distinguish between individual modes based upon their natural frequency. These issues will not be considered in the present study, since the intention here is to demonstrate that multivariate emulators can be applied to the uncertain FE problem in its simplest form, without introducing additional levels of complexity.

3. Emulators

Monte Carlo uncertainty analysis requires the model to be run at many input configurations in order to make accurate inference about the uncertainty distribution, and the number of runs required increases exponentially with the number of